EMAT 102

B.TECH. IInd SEMESTER EXAMINATION, 2023-24

B.TECH.

(Engineering Mathematics-II)

(CSE, ME, IT, ECE)

(CBCS Mode)

AFFIX PRESCRIBED RUBBER STAMP

Date (तिथि) : ____________________

Paper ID
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6014

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Time : 1:30 Hrs.
समय : 1:30 घण्टे

Max. Marks : 75
अधिकतम अंक : 75

नोट : पुस्तिका में 50 प्रश्न दिये गये हैं, सभी प्रश्न करने होंगे। प्रत्येक प्रश्न 1.5 अंक का होगा।

Important Instructions :

1. The candidate will write his/her Roll Number only at the places provided for, i.e. on the cover page and on the OMR answer sheet at the end and nowhere else.

2. Immediately on receipt of the question booklet, the candidate should check up the booklet and ensure that it contains all the pages and that no question is missing. If the candidate finds any discrepancy in the question booklet, he/she should report the invigilator within 10 minutes of the issue of this booklet and a fresh question booklet without any discrepancy be obtained.

महत्वपूर्ण निर्देश :

1. अभ्यर्थी अपने अनुक्रमांक केवल उन्हीं स्थानों पर लिखेंगे जो इसके लिए दिये गये हैं, अर्थात् प्रश्न पुस्तिका के मुख्य पृष्ठ तथा साथ दिये गये ओ०एम०आर० उत्तर पत्र पर, तथा अन्यत्र कहीं नहीं लिखेंगे।

2. प्रश्न पुस्तिका मिलते ही अभ्यर्थी को जाँच करके सुनिश्चित कर लेना चाहिए कि इस पुस्तिका में पूरे पृष्ठ हैं और कोई प्रश्न छूटा तो नहीं है। यदि कोई विसंगति है तो प्रश्न पुस्तिका मिलने के 10 मिनट के भीतर ही कक्ष परिप्रेक्षक को सूचित करना चाहिए और बिना त्रुटि की दूसरी प्रश्न पुस्तिका प्राप्त कर लेना चाहिए।

1. If $F(t) = \text{sinh}at$, then $L[F(t)]$ is :

(A) $\frac{s}{s^2-a^2}$

(B) $\frac{1}{s^2+a^2}$

(C) $\frac{a}{s^2-a^2}$

(D) $\frac{a}{s^2+a^2}$

2. If n be a matural number, then Laplace transform of $t^n e^{-at}$ is :

(A) $\frac{\lfloor n}{(s+a)^n}$

(B) $\frac{\lfloor n}{(s-a)^n}$

(C) $\frac{\lfloor n}{(s+a)^{n+1}}$

(D) $\frac{\lfloor n}{(s-a)^{n+1}}$

$\frac{\lfloor 2}{S+a}$

$t^n \quad e^{-at}$

3. The one-dimensional wave equation $\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial y^2}$ is :

(A) Hyperbolic

(B) Elliptic

(C) Parabolic

(D) None of these

4. If $L[F(t)] = f(s)$, then $L[F''(t)]$ is :

(A) $s^2 f(s) - SF'(0) - F(0)$

(B) $s^2 f(s) - SF(0) - F'(0)$

(C) $s^2 f(s) - F(0)$

(D) $s^2 f(s) - SF'(0) + F(0)$

5. If we consider the linear partial differential equation of second order i.e $R_r + S_s + T_t + f(x, y, z, p, q) = 0$ Then it is called parabolic if :

(A) $s^2 - 4RT > 0$

(B) $s^2 > 4RT$

(C) $s^2 - 4RT = 0$

(D) None of these

6. If $F(t) = a + bt + \frac{c}{\sqrt{t}}$, then $L[F(t)]$ is :

(A) $\frac{a}{s} + \frac{b}{s^2} + c\sqrt{\frac{\pi}{s}}$

(B) $\frac{a}{s^2} + \frac{b}{s} + c\sqrt{\frac{\pi}{s}}$

(C) $\frac{a}{s} + \frac{b}{s^2} + c\sqrt{\frac{s}{\pi}}$

(D) $\frac{a}{s} + \frac{b}{s^2} + \frac{c\pi}{\sqrt{s}}$

$c(t)^{-1/2}$ $\frac{\sqrt{\frac{1}{2}}}{s^{1/2}}$

$\frac{a}{s} + \frac{b}{s^2} + c\sqrt{\frac{\pi}{s}}$

7. A first order partial differential equation $xp + yq = xz^2$ is :

(A) Semi-linear

(B) Linear

(C) Quasi-linear

(D) Non-linear

8. The value of $\int_0^\infty e^{-2t} \cos 3t \, dt$ is :

(A) $\frac{3}{13}$

(B) $\frac{1}{13}$

(C) $\frac{4}{13}$

(D) $\frac{2}{13}$

$e^{-2t} \int \cos 3t - \int e^{-2t} \cdot 2 \frac{\sin 3t}{3}$

$e^{-2t} \frac{s}{s^2 + 9}$

9. The order and degree of partial differential equation $\left(\frac{\partial z}{\partial x}\right)^3 + \left(\frac{\partial z}{\partial y}\right)^4 = z$ are :

(A) 1, 3

(B) 1, 4

(C) 2, 3

(D) 2, 4

10. If $F(t) = \int_0^t \int_0^t \int_0^t \cos au \, du \, du \, du$, then $L[f(t)]$ is :

(A) $\frac{1}{s(s^2+a^2)}$

(B) $\frac{1}{s^2(s^2+a^2)}$

(C) $\frac{1}{s^3(s^2+a^2)}$

(D) $\frac{s}{(s^2+a^2)}$

$\frac{s}{s^2+a^2}$

11. The differential equation $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\{\log(x+1)\}$ is :

(A) Cauchy-Euler equation

(B) Simultaneous linear differential equation

(C) None of these

(D) Legendre's linear differential equation

12. If $f(s) = \frac{s+5}{(s+1)(s+3)}$ then $L^{-1}\{f(s)\}$ is :

(A) $2e^{-t} - e^{-3t}$

(B) $2e^{-t} + e^{-3t}$

(C) $e^{-t} + 2e^{-3t}$

(D) $e^{t} + 2e^{-3t}$

e^{-3t}

13. Solution set of the equation $\frac{dx}{yz} = \frac{dy}{zx} = \frac{az}{xy}$ is :

(A) $x^2 + y^2 = c_1 \;\; y^2 - z^2 = c_1$

(B) $x^2 - y^2 = c_1 \;\; x^2 - z^2 = c_2$

(C) $x^2 - y^2 = c_1 \;\; x^2 + z^2 = c_2$

(D) None of these

14. If $L^{-1}\{f(s)\} = F(t)$ then $L^{-1}\{f(as)\}$ is :

(A) $aF(at)$

(B) $F(at)$

(C) $F(t/a)$

(D) $\frac{1}{a} F(t/a)$

15. solution set of equation $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$ is :

(A) $xy = c_1, \; yz = c_2$

(B) $\frac{x}{y} = c_1, \; yz = c_2$

(C) $\frac{x}{z} = c_1, \; \frac{y}{z} = c_2$

(D) $xy = c_1, \; \frac{y}{z} = c_2$

16. If $f(s) = \log\left(1 + \frac{1}{s}\right)$ then $L^{-1}\{f(s)\}$ is :

(A) $\frac{1}{t} + \frac{e^{-t}}{t}$

(B) $\frac{1-e^{t}}{t}$

(C) $\frac{1-e^{-t}}{t}$

(D) $\frac{1+e^{t}}{t}$

17. $y = e^x$ is a solution of differential equation $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0$ if :

(A) $1 + P + Q = 0$

(B) $1 - P + Q = 0$

(C) $-1 + P + Q = 0$

(D) $P + Q = 0$

18. If $L^{-1}\{f(s)\} = F(t)$ and $L^{-1}\{g(s)\} = G(t)$ then $L^{-1}[f(s).g(s)]$ is :

(A) $\int_0^t F(u)\, G(t-u)\, dy$

(B) $\int_0^t F(u)\, G(t+u)\, dy$

(C) $\int_0^t F(u)\, G(u)\, dy$

(D) $\int_0^t F(t-u)\, G(t+u)\, dy$

19. $y = x^m$ is a solution of differential equation $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0$ if :

(A) $m^2 + m\,P + Q = 0$

(B) $m(m^2 - 1) + Pm + Qx^2 = 0$

(C) $m^2(m-1) + Pmx + Qx^2 = 0$

(D) $m(m-1) + Pmx + Qx^2 = 0$

20. If $f(s) = \frac{1}{s(s^2+a^2)}$ then $L^{-1}\{f(s)\}$ is :

(A) $\frac{1}{a^2}(1 - \cos at)$

(B) $\frac{1}{a^2}(1 + \cos at)$

(C) $\frac{1}{a^2}(\cos at - 1)$

(D) $-\frac{1}{a^2}(1 + \cos at)$

$\frac{1}{s} + \frac{1}{s^2+a^2}$

$1 + \cos at$

21. The complementary function of the differential equation $(x^2D^2 - xD - 3)y = x^2 \log x$ is :

$m^2 - m - 3$

$1 \pm \sqrt{1-}$

(A) $c_1 \cos x + c_2 \sin x$

(B) $\frac{c_1}{x} + c_2\, x^3$

(C) $c_1 \log x + c_2\, x^3$

(D) $\frac{c_1}{x} + c_2\, (\log x)^3$

so $(s)^{1/2}$

$\frac{1}{2\sqrt{s}}$

22. If $f(s) = \frac{1}{\sqrt{s}}$ then $L^{-1}\{f(s)\}$ is :

(A) $t^{-1/2}$

(B) $t^{1/2}$

(C) $\frac{1}{\sqrt{\pi t}}$

(D) $\sqrt{\pi t}$

$t L^{-1}[F(t)] = \frac{1}{2}[\sqrt{s}]$

$\frac{1}{2t}$

P.I. $= \frac{e^x}{D^2 - 3D + 2}$

$= \frac{x e^x}{2D - 3}$

$= \frac{-x e^x}{1}$

23. The general solution of differential equation $(D^2 - 3D + 2)\, y = e^x$ is :

(A) $y = c_1 e^x + c_2 e^{2x} - x e^x$

(B) $y = c_1 e^x + c_2 e^{-2x} - x e^x$

(C) $y = c_1 e^x + c_2 e^{2x} + x e^x$

(D) $y = c_1 e^x + c_2 e^{3x} + x$

$m^2 - 3m + 2 = 0$

$m^2 - 2m - m + 2 = 0$

$m(m-2) - 1(m-2) = 0$

$(m-2)(m-1) = 0$

$m = 2,\ m = 1$

$c_1 e^x + c_2 e^{2x}$

24. If $F(t) = \begin{cases} 1, & 0 < t \leq 1 \\ 0, & t > 1 \end{cases}$ then $L[P(t)]$ is :

(A) $\frac{1+e^{-s}}{s}$

(B) $\frac{1-e^{-s}}{s}$

(C) $\frac{1+e^{+s}}{s}$

(D) Does not exist

$\int_0^1 1$

$c_1 e^x + c_2 e^{2x} - x e^x$

e^{-s}

$\frac{1}{s}$

25. The particular integral of a differential equation $(D^2 + 4)\, y = \cos 2x$ is :

(A) $\frac{1}{4}\cos 2x$

(B) $\frac{x}{2}\cos 2x$

(C) $\frac{x}{4}\ sin\ 2x$

(D) $\frac{x}{4}\cos 2x$

$y = \frac{\cos 2x}{D^2+4}$

26. The function $f(x) = \cos 5x$ is a periodic function having fundamental period :

(A) π

(B) 2π

(C) 10π

(D) $\frac{2\pi}{5}$

$= 10 \cdot$

27. If the roots of auxillary equation of a differential equation are $1, 1 \pm i$, then the complementary function corresponding to the roots is :

(A) $c_1\, e^x + x c_2 e^x + c_3 \cos x + c_4 \sin x$

(B) $c_1\, e^x + c_2 \cos x + c_3 \sin x$

(C) $(c_1 + c_2 x)\, e^x + (c_3 + c_4 x) \sin x$

(D) None to these

$c_1 e^x + c_1$

28. The function $f(x) = \frac{sin n\pi x}{l}$ is a periodic function having fundamental period :

(A) π

(B) $\frac{2l}{n}$

(C) $2l$

(D) 2π

29. The value of $\frac{1}{D-a}\, f(x)$ is obtained by :

(A) $e^{-ax} \int e^{ax} f(x) dx$

(B) $e^{-ax} \int e^{-ax} f(x) dx$

(C) $e^{ax} \int e^{ax} f(x) dx$

(D) $e^{ax} \int e^{-ax} f(x) dx$

30. If $L[F(t)] = f(s)$ then $L[tF(t)]$ is :

(A) $f'(s)$

(B) $-f'(s)$

(C) $f'(s) - F(o)$

(D) $sf'(s) - F(o)$

31. The value of $sin n\pi$ is :

(A) 1 for all n

(B) $(-1)^n$

(C) 0 for all n

(D) None of these

32. The particular integral of differential equation $(D^2 - 2KD + K^2)\, y = e^x$ is :

(A) e^x

(B) $\frac{e^x}{1-K}$

(C) $\frac{e^x}{(1-K)^2}$

(D) $e^x(1-K)^2$

P.I. $= \frac{e^x}{D^2-2KD+K^2}$

$= \frac{e^x}{1-2+K^2}$

33. If $f(x)$ and $g(x)$ are even function, then :

(A) $f(x).\, g(x)$ is an odd function

(B) $f(x).\, g(x)$ is an even function

(C) $f(x)\; g(x)$ is neither even nor odd function

(D) None of these

$D(D-1)+a^2y=0$

34. The solution of differential equation $\frac{d^2y}{dx^2} + a^2y = 0$ is :

(A) $y = c_1e^{-ax} + c_2e^{ax}$

(B) $y = c_1 \cos ax + c_2 \sin ax$

(C) $y = c_1 \cos ax + \sin ax$

(D) $y = c_1e^{-ax} + e^{ax}$

$D + a^2y = 0.$

$y = \cancel{C_1 e^{-x}}$

$D^2 + Da^2y = 0$

$D(D+a^2)\,y = 0$

35. If $f(x)$ is a periodic function defined over $-\pi < x < \pi$ then, in the Fourier series representation, the value of a_n is obtained by :

(A) $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, cosnx\, dx$

(B) $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, \frac{sinn\pi x}{l}\, dx$

(C) $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, sinnx\, dx$

(D) $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, \frac{cosn\pi x}{al}\, dx$

36. Homogeneous linear differential equation $x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 4y = 0$ will reduce to linear differential equation with constant coefficient by putting :

(A) $x = te^t$

(B) $x = \log t$

(C) $x = e^t$

(D) $x = e^{t^2}$

37. If $f(x) = x$ is expanded in a Fourier sin e series in (o, π), then b_n is equal to :

(A) $\frac{1}{n}$

(B) $\frac{2}{n}(-1)^n$

(C) $\frac{2}{n}(-1)^{n+1}$

(D) $\frac{4}{n}(-1)^n$

$\int_0^{\pi}$ $b_n = \frac{1}{\pi}\int_0^{\pi} f(x) \cos$

$\frac{1}{\pi}\int_0^{\pi} x \cos nx$

$\frac{1}{\pi}\left[x \int \cos nx - \right.$

$\frac{1}{\pi}\left[\frac{x}{n} \sin nx + \right.$

$\frac{1}{\pi n}\left[x \sin nx - \frac{1}{n} \right.$

$\frac{1}{\pi n}\left[\cos nx \right]^{\pi}$

$(-1)^n -$

38. The degree of differential equation $\frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = 0$ is :

(A) 1

(B) 2

(C) 3

(D) Not defined

39. If $f(x) = |x|$ is expanded in a Fourier series in $(-1, 1)$ then a_o is equal to :

(A) 1/2

(B) 2

(C) 1

(D) None of these

$a_0 = \frac{1}{\pi}\int_{-1}^{1} f(x)\,dx$

$= \frac{2}{\pi} \times \left[\frac{x^2}{2}\right]_0^1 = \frac{2}{\pi} \times \frac{1}{2}$

40. If $f(x) = 1 + x$ is expanded in a Fourier series in (-3, 3), then the value of a_n for $n = 1, 2, 3, ...$ is :

(A) 0

(B) 1

(C) 2

(D) $\frac{2}{n}$

$\int (1+x)$

$\frac{1}{3}\int_{-3}^{3} (1+x) \cos n\pi x\,dx$

$\frac{2}{3}\int_{0}^{3} (1+x) \cos n\pi x\,dx$

41. If $f(t) \begin{cases} \sin(t - 2\pi), & t > 2\pi \\ 0, & t < 2\pi \end{cases}$ then $L[F(t)]$ is :

(A) $e^{-2\pi s}\frac{1}{s^2+1}$

(B) $e^{2\pi s}\frac{1}{s^2+1}$

(C) $e^{\pi s}\frac{1}{s^2+1}$

(D) $e^{-\pi s}\frac{1}{s^2+1}$

42. If $f(x) = x^2, -\pi < x < \pi$, then in the Fourier series expansion the value to a_o is :

(A) $\frac{2\pi^2}{3}$

(B) $2\pi^2$

(C) $\frac{2\pi}{3}$

(D) $\frac{\pi^2}{3}$

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\,dx$

$= \frac{2}{\pi}\int_{0}^{\pi} x^2 \quad \frac{2}{\pi}\left[\frac{x^3}{3}\right]_0^{\pi}$

$\frac{2}{\pi} \times \frac{\pi^3}{3}$

43. The differential equation, which involves two or more than two independent variables is called :

(A) Partial differential equation

(B) Ordinary differential equation

(C) Exact differential equation

(D) Total differential equation

44. The degree of differential equation $x\frac{dy}{dx} + \frac{2}{(dy/dx)} = y^2$ is :

(A) 2

(B) 1

(C) 3

(D) Does not defined

45. If $F(t) = e^{-2t} sin4t$, then $L[F(t)]$ is :

(A) $\frac{2}{s^2+4s+20}$

(B) $\frac{s-2}{s^2+4s+20}$

(C) $\frac{s-4}{s^2+4s+20}$

(D) $\frac{4}{s^2+4s+20}$

$\frac{4}{s^2+16}$

$\frac{4}{(s-2)^2+16}$

46. The function $f(x) = sinx + cos2x + sin3x$ is a periodic function with period :

(A) π

(B) 2π

(C) $= 3\pi/2$

(D) $= 2\pi/3$

47. If $f(x)$ be any function defined in the interral $c < x < c + 2l$, then the Fourier series of $f(x)$ is :

(A) $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} - b_n \sin\left(\frac{n\pi x}{l}\right)$

(B) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + b_n \sin n\pi x$

(C) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l}$

(D) None of these

48. The order of differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$ is :

(A) 3

(B) 1

(C) 2

(D) 4

$\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}$

49. The general solution of second order differential equation contains :

(A) One arbitrary constant

(B) Two arbitrary constant

(C) There arbitrary constant

(D) No arbitrary const

50. If $f(x) = \begin{cases} -1, & -\pi < n < 0 \\ 1 & 0 < x < \pi \end{cases}$ then the Fourier series of $f(x)$ is :

(A) $f(x) = \frac{1}{\pi}\left[sinx + \frac{sin\,3x}{3} + \frac{sin\,5x}{5} + \cdots\right]$

(B) $f(x) = \frac{4}{\pi}\left[sinx + \frac{sin\,3x}{3} + \frac{sin\,5x}{5} + \cdots\right]$

(C) $f(x) = 4\left[sinx + \frac{sin\,3x}{3} + \frac{sin\,5x}{5} + \cdots\right]$

(D) $f(x) = \frac{4}{\pi}\left[cosx + \frac{cos\,3x}{3} + \frac{cos\,5x}{5} + \cdots\right]$

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