EMAT 101

B. Tech. Ist SEMESTER EXAMINATION, 2024-25

B.TECH.

Engineering Mathematics-I

(CSE, ME, ECE & IT)

(CBCS Mode)

AFFIX PRESCRIBED RUBBER STAMP

Date (तिथि) : ____________

Paper ID
(To be filled in the OMR Sheet)

2134

अनुक्रमांक (अंकों में) :
Roll No. (In Figures) :

अनुक्रमांक (शब्दों में) :

Roll No. (In Words) : ____________

Time : 1:30 Hrs.
समय : 1:30 घण्टे

Max. Marks : 75
अधिकतम अंक : 75

नोट : पुस्तिका में 50 प्रश्न दिये गये हैं, सभी प्रश्न करने होंगे। प्रत्येक प्रश्न 1.5 अंक का होगा।

Important Instructions :

1. The candidate will write his/her Roll Number only at the places provided for, i.e. on the cover page and on the OMR answer sheet at the end and nowhere else.

2. Immediately on receipt of the question booklet, the candidate should check up the booklet and ensure that it contains all the pages and that no question is missing. If the candidate finds any discrepancy in the question booklet, he/she should report the invigilator within 10 minutes of the issue of this booklet and a fresh question booklet without any discrepancy be obtained.

महत्वपूर्ण निर्देश :

1. अभ्यर्थी अपने अनुक्रमांक केवल उन्हीं स्थानों पर लिखेंगे जो इसके लिए दिये गये हैं, अर्थात् प्रश्न पुस्तिका के मुख्य पृष्ठ तथा साथ दिये गये ओ०एम०आर० उत्तर पत्र पर, तथा अन्यत्र कहीं नहीं लिखेंगे।

2. प्रश्न पुस्तिका मिलते ही अभ्यर्थी को जाँच करके सुनिश्चित कर लेना चाहिए कि इस पुस्तिका में पूरे पृष्ठ हैं और कोई प्रश्न छूटा तो नहीं है। यदि कोई विसंगति है तो प्रश्न पुस्तिका मिलने के 10 मिनट के भीतर ही कक्ष परिप्रेक्षक को सूचित करना चाहिए और बिना त्रुटि की दूसरी प्रश्न पुस्तिका प्राप्त कर लेना चाहिए।

1. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

(A) 1

(B) 0

(C) 3

(D) 2

2. The value of x for which the rank of matrix $\begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$ is 2 ?

(A) 3

(B) 2/5

(C) 3/5

(D) 2

$2(x) - 4(3x-2) + 2(3$

$2x - 12x + 8 - 2 = 2$

$-10x + 6 = 2 - 6$

$-10x = 4$

$x = -\frac{4}{10} = \frac{2}{5}$

3. The system of equations

$x + y + z = -3$

$3x + y - 2z = -2$

$2x + 4y + 7z = 7$ is

(A) Consistent and has unique solution

(B) Inconsistent and has no solution

(C) Consistent and has infinite solutions

(D) Consistent with 2 solutions

4. The value of λ and μ for which the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ has infinite solutions are respectively -

(A) 3,6

(B) 1, 10

(C) 1, 6

(D) 3, 10

$1(20-12) - 2(5-4) + 3(6-4)$

5. The eigen values of the matrix $A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 0 \\ 3 & 1 & -3 \end{bmatrix}$ are

(A) 3, -5, 2

(B) 1, -5, 2

(C) 3, 5, 2

(D) 3, -5, -2

$\lambda^3 + 0 + |A|$

6. Which of the following is true for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

(A) $A^3 + 6A^2 + 9A + 4I = 0$

(B) $A^3 - 6A^2 - 9A - 4I = 0$

(C) $A^3 - 6A^2 + 9A - 4I = 0$

(D) $A^3 - 3A^2 + 6A - 4I = 0$

7. The inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ is -

(A) $\frac{1}{5}\begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$

(B) $\frac{-1}{5}\begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$

(C) $\frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

(D) $\frac{-1}{5}\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix}$

$|A| = -1 - 4 = -5$

$A' = \begin{bmatrix} 1 & -2 \\ -2 & -1 \end{bmatrix}^T$

$= -\frac{1}{5}\begin{bmatrix} 1 & -2 \\ -2 & -1 \end{bmatrix}^T$

$-\frac{1}{5}\begin{bmatrix} 1 & -2 \\ -2 & -1 \end{bmatrix}$

8. If $\tan(\theta + i\phi) = cos\alpha + isin\alpha$, then the value of θ is -

(A) $\frac{n\Pi}{2}$

(B) $\frac{\Pi}{4}$

(C) $\frac{n\Pi}{2} + \frac{\Pi}{4}$

(D) $\frac{n\Pi}{2} - \frac{\Pi}{4}$

$\begin{bmatrix} 1 & -2 \\ -2 & -1 \end{bmatrix}^T$

$-\frac{1}{5}\begin{bmatrix} 1 & -2 \\ -2 & -1 \end{bmatrix}$

λ^3 (trace of A))λ^2 + (...

9. The value of $Log_e^{(-i)}$ is -

(A) $-i\Pi/2$

(B) $i\Pi/2$

(C) $i\Pi/2 + 2n\Pi i$

(D) $-i\Pi/2 + 2n\Pi i$

10. The value of i^i are -

(A) $e^{-(4n+1)\frac{\Pi}{2}}$

(B) $e^{(4n+1)\Pi/2}$

(C) $e^{(4n-1)\Pi/2}$

(D) None of these

11. If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ be a position vector and r be its magnitude then div r is -

(A) 0

(B) 2

(C) 4

(D) 3

12. The value of $\int_0^\infty x^{1/4} e^{-\sqrt{x}} dx$ is -

(A) $\frac{3}{2}\sqrt{\pi}$

(B) $\sqrt{\pi}$

(C) $\frac{1}{2}\sqrt{\pi}$

(D) $\frac{5}{2}\sqrt{\pi}$

13. The value of $\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}}\,dx$ using Beta-Gamma function is -

(A) $\beta(9,15)$

(B) $\beta(15,9)$

~~(C)~~ 0

(D) $\beta(8,6)$

14. $\int_0^{\pi/2} \sqrt{tan\theta}\, d_\theta$ is equal to -

(A) $\sqrt{\pi}/2$

(B) $\pi/2$

(C) $\sqrt{2}\pi$

~~(D)~~ $\frac{\pi}{\sqrt{2}}$

15. The real part of $sec\,(x+iy)$ is -

(A) $\frac{2\sin\alpha\sinh\beta}{\cos 2\alpha+\cosh 2\beta}$

~~(B)~~ $\frac{2\cos\alpha\cosh\beta}{\cos 2\alpha+\cosh 2\beta}$

(C) $\frac{2\sinh\alpha\sin\beta}{\cos 2\alpha+\cosh 2\beta}$

(D) $\frac{2\cosh\alpha\cos\beta}{\cos 2\alpha+\cosh 2\beta}$

16. The value of $\int_0^1\int_0^x e^x dx dy$ is -

(A) 1

~~(B)~~ -1

~~(C)~~ e

(D) 1/e

17. By changing into polar coordinates $\int_0^a \int_0^{\sqrt{a^2-y^2}} y^2 \sqrt{x^2+y^2}\,dx\,dy$ is equal to-

$\left[\frac{3}{2}\frac{y^2}{y}(x^2+y^2)^{3/2}\right]$ so

(A) πa^5

(B) 0

(C) $\frac{\pi a^5}{20}$

(D) πa^4

18. The real part of $e^{e^{i\theta}}$ is -

(A) $e^{\cos\theta}\cos(\sin\theta)$

(B) $e^{\cos\theta}\sin(\cos\theta)$

(C) $e^{\cos\theta}\cos(\cos\theta)$

(D) $e^{\cos\theta}\sin(\sin\theta)$

19. If $u = x^2yz - 4y^2z^2 + 2xz^3$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ is equal to -

(A) u

(B) $2u$

(C) $3u$

(D) $4u$

$x(2xyz + 2z^3) + y(x^2z - 8yz^2)$

$x(2xyz + 2z^3) + y(x^2z - 8yz^2) + 2($

$2(x^2y - 8$

$2x^2yz + 2xz^3 + yx^2z - 8y^2z$

$+ 2x^2y - 8y^2z^2 + 6z^3x$

$8xz^3 + 3x^2yz$

20. The period of e^z is -

(A) π

(B) 2π

(C) πi

(D) $2\pi i$

21. The value of $\int_0^1 \int_0^1 \frac{dx\,dy}{\sqrt{(1-x^2)(1-y^2)}}$ is -

(A) $\pi^2/2$

(B) $\pi/2$

(C) $\pi^2/4$

(D) $\pi/4$

$\frac{1}{\sqrt{(1-x^2)(1-y)^2}} = \frac{A}{(1-x^2)} + \frac{B}{(1-y^2)}$

$= A -$

22. If $x = r\cos\theta, y = r\sin\theta, z = z$ then the Jacobian $\frac{\partial(r,\theta.z)}{\partial(x,y,z)}$ is equal to -

(A) r

(B) $1/r$

(C) r^2

(D) $1/_{r^2}$

23. 20^{th} derivative of the function $y = \underline{\sin 2x \sin 3x}$ is -

$\sin 2x \sin 3x = \frac{1}{2}\cos(a-b) -$

Cop

(A) $\frac{1}{2}[\cos x - 5^{20}\cos 5x]$

(B) $\frac{1}{2}[\cos 5x - 5^{20}\cos x]$

(C) $\frac{1}{2}[\sin x - 5^{20}\sin 5x]$

(D) $\frac{1}{2}[\sin 5x - 5^{20}\sin x]$

24. The matrix $A = \begin{bmatrix} 2 & 3-4i \\ 3+4i & 2 \end{bmatrix}$ is -

(A) Normal

(B) Unitary

(C) Hermitian

(D) Skew Hermitian

25. The nullity of the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ is –

(A) 2

(B) 1

(C) 3

(D) 0

26. Is the function $f(x,y) = x^2 + y^2 + 6x + 12$ at a given stationary point (-3,0) is -

(A) Maxima

(B) Minima

(C) Neither maxima nor minima

(D) Both maxima and minima

27. If the expansion of $e^x siny$ in powers of x and y. The coefficient x^3 is -

(A) 1

(B) 0

(C) $\frac{1}{2}$

(D) $^1/_6$

28. Integral $\int_0^3 \int_0^1 (x^2 + 3y^2) dy\, dx$ is equal to -

(A) 4

(B) 12

(C) 8

(D) 16

29. The value of $\sqrt{\frac{1}{3}}\sqrt{\frac{2}{3}}$ is-

(A) $\sqrt{2}\pi$

(B) 2π

(C) $^{2\pi}/_{\sqrt{3}}$

(D) π

30. The curve, which equation is $x^{2/3} + y^{2/3} = a^{2/3}$ is symmetrical about the line-

(A) x-axis

(B) y-axis

(C) Both x-axis and y-axis

(D) Neither x-axis nor y-axis

31. If the eigen value of a square matrix A are $1, -1, 2$. Then $|A|$ is equal to -

(A) 1

(B) 2

(C) -2

(D) None of these

32. If $u = \log_e\left(\frac{x^4+y^4}{x+y}\right)$, then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to -

(A) $3u$

(B) 3

(C) u

(D) None of the above

33. If $z = e^{ax+by} f(ax - by)$, then $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y}$ is equal to -

(A) $2abz$

(B) abz

(C) $(a+b)z$

(D) $(a-b)z$

34. The diagonal form of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ is -

(A) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & \sqrt{5} & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix}$

35. The value of λ for which the vectors $(1,-2,\lambda),(2,-1,5)$, and $(3,-5,7\lambda)$ are linearly dependent.

$$\begin{vmatrix} 1 & -2 & \lambda \\ 2 & -1 & 5 \\ 3 & -5 & 7\lambda \end{vmatrix}$$

$1(-\lambda+25)+2(14\lambda-15)+\lambda$

$\cancel{25-\lambda+28\lambda-30} = 10\lambda+3\lambda$

$20\lambda-$ "

(A) $-\frac{3}{14}$

(B) $\frac{3}{7}$

(C) $\frac{5}{7}$

(D) $\frac{5}{14}$

36. The value of $\sin(\log_e i^i)$ is -

(A) 1

(B) -1

(C) 0

(D) i

37. If $x+y+z=u,\ y+z=uv,\ z=uvw$, then the value of $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ is -

(A) $-u^2v$

(B) $1/_{u^2v}$

(C) u^2v

(D) $-1/_{u^2v}$

38. The degree of homogeneous function $f(x,y)=\frac{x^{1/4}+y^{1/4}}{x^{1/5}+y^{1/5}}$ is -

(A) $1/_4$

(B) $1/_5$

(C) $1/_{20}$

(D) 0

$\frac{x^{1/4}}{x^{1/5}}$ $x^{1/4-1/5}$ $\frac{1}{4}-\frac{1}{5}$ $\frac{5-1}{20}$

39. If $y_1 = cosx_1, y_2 = sinx_1 cosx_2$ and $y_3 = sinx_1, sinx_2 cosx_3$ then $\frac{\partial(y_1,y_2,y_3)}{\partial(x_1,x_2,x_3)}$ is equal to -

(A) $-sin^3 x_1\, sin^2 x_2\, sinx_3$

(B) $-sinx_1\, sin^2 x_2\, sin^3 x_3$

(C) $cos^3 x_1\, cos^2 x_2\, cosx_3$

(D) $-cos^3 x_1\, cos^2 x_2\, cosx_3$

40. If A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a square matrix A. Then the characteristic eqn of matrix A is -

(A) $\lambda^2 + (trace\ A)\lambda + |A| = 0$

(B) $\lambda^2 - (trace\ A)\lambda - |A| = 0$

(C) $\lambda^2 + (trace\ A)\lambda - |A| = 0$

(D) $\lambda^2 - (trace\ A)\lambda + |A| = 0$

41. The value of $\int_0^{\Pi/2} \int_0^{a\cos\theta} r\sin\theta\, dr d\theta$ is -

$\left[r^2 \sin^2\theta\right]$

$\frac{a^2}{2}\int\left[a\cos^2\theta \sin^2\theta\right]$

$\frac{a^2}{2}\left[\right.$

(A) $\frac{a^2}{4}$

(B) $\frac{a^2}{6}$

(C) $\frac{a}{4}$

(D) $\frac{a}{6}$

42. Differentiating n times the differential equation $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ is -

(A) $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$

(B) $(1-x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$

(C) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2-1)y_n = 0$

(D) $(1-x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2-1)y_n = 0$

43. If $y = x^n \log x$, then the value of y_{x+2} is -

y_{n+2}

$x^n \log x$

(A) $\frac{|\underline{n}}{x^2}$

~~(B)~~ $\frac{-|\underline{n}}{x^2}$

y_{n+2}

(C) $\frac{|\underline{n}}{x}$

(D) $\frac{-|\underline{n}}{x}$

$\cos^3 x$

$3\cos^2 x \cdot \sin x$

$\cos^3 x$

44. The 100[th] derivative of $y = cos^3 x$ is -

~~(A)~~ $\frac{3}{4}[3^{99} cos 3x + cos x]$

(B) $\frac{1}{2}[3^{100} cos 3x + cos x]$

(C) $\frac{1}{4}[3^{99} cos x + cos 3x]$

(D) $\frac{1}{4}[3^{100} cos x + 3cos 3x]$

45. If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ be the position vector and r be its magnitude, then grad r^n is equal to -

(A) $n\, r^{n-2}$

(B) $n\, r^{n-1}\vec{r}$

(C) $n\, r^{n-2}\vec{r}$

~~(D)~~ $n\, r^{n-2}\hat{r}$

46. If A be a Hermitian matrix, then iA is -

(A) Hermitian matrix

~~(B)~~ Skew- Hermitian matrix

(C) Both Hermitian and skew- Hermitian

(D) None of these

47. If u is homogeneous function of degree n in x & y.

Then $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$ is equal to -

~~(A)~~ $(n^2 - n)u$

(B) nu

(C) $(n-1)u$

(D) $n(n+1)u$

$n(n-1)u$

48. If $x = e^v sec u, y = e^v \tan u$ then the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ is equal to -

(A) $e^{-2v} \sec u$

~~(B)~~ $e^{2v} \sec u$

(C) $e^{-2v} \cos u$

~~(D)~~ $-e^{-2v} \cos u$

49. A condition of minima of a stationary point of function $z = f(x,y)$ is $\left(r = \frac{\partial^2 f}{\partial x^2}, \; s = \frac{\partial^2 f}{\partial x \partial y}, \; t = \frac{\partial^2 f}{\partial y^2}\right)$

(A) $rt - s^2 > 0, \; r > 0$

~~(B)~~ $rt - s^2 < 0, \; r > 0$

~~(C)~~ $rt - s^2 > 0, \; r < 0$

(D) $rt - s^2 < 0, \; r < 0$

50. The corresponding upper triangular matrix for the matrix $\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ is -

(A) $\begin{bmatrix} -1 & 2 & -2 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

~~(B)~~ $\begin{bmatrix} -1 & 2 & -2 \\ 0 & 4 & -1 \\ 0 & 0 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 2 & -2 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} -1 & 2 & -2 \\ 0 & 4 & -1 \\ 0 & 0 & 5/4 \end{bmatrix}$

$\begin{bmatrix} -1 & 2 & -2 \\ 0 & 1 & 1 \\ -1 & -1 & 0 \end{bmatrix}$

$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$

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