EMAT 103

B. Tech. I[st] SEMESTER EXAMINATION, 2025-26

BACHELOR OF TECHNOLOGY

(CSE)

Mathematics for Data Science

8947

Time : Three Hours] [Maximum Marks : 75

Note: There are **three** sections (**A**, **B** and **C**) and candidate has to attempt questions from all sections. Marks are indicated against each section.

Section-A

1. Attempt all parts of the following : 5×3=15

   (a) Find $\int \frac{(\log x)^2}{x} dx$.

   (b) State Cauchy's mean value theorem (CMVT).

   (c) If $f: R \to R$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.

   (d) Expand sin x in powers of $\left(x - \pi/_2\right)$.

   (e) Let $f(x,y) = ln(x^2 + y^2)$, then show that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.

Section-B

Note: Attempt **all** questions : 4×5=20

2. (a) If sets $A = \{1,3\}, B = \{2,4\}$ *and* $C = \{3,4\}$, then find

   (i) $A \times (BUC)$

   (ii) $(A \times B) \cap (A \times C)$

Or

(b) Let $f(x) = x^2 + 2$ and $g(x) = \frac{x}{x-1}$ are two real value functions. Find fog, gof and also show that $fog \neq gof$.

3. (a) Evaluate $\int \frac{x+2}{2x^2+6x+5} dx$.

Or

(b) Verify the Rolle's theorem for the function $f(x) = x^2 - 4x + 3$ on [1,3].

4. (a) Trace the curve $y(1 - x^2) = 1$.

Or

(b) Express $\int_0^1 x^m(1 - x^p)^n \, dx$ in term of beta function and hence evaluate $\int_0^1 x^5(1 - x^3)^{10} dx$.

5. (a) Define order and degree of differential equation. Determine the order and degree of the differential equation :

$$\sqrt{y + \left(\frac{dy}{dx}\right)^2} = 1 + x.$$

Or

(b) Show that the radius of curvature to the curve $y = 6x + 5x^2 + x^3$ at origin is $\frac{37\sqrt{37}}{10}$.

Section-C

Note: Attempt any two questions : 2×20=40

6. (a) Explain the following in one independent variable.

   (i) Function

   (ii) Limit

   (iii) Continuity

   (iv) Differentiability

   (b) A function $f(x)$ is defined as follows :

   $$f(x) = \begin{cases} 1 + x, & x \leq 2 \\ 5 - x, & x > 2 \end{cases}$$

   Show that $f(x)$ is not differentiable at $x = 2$

7. (a) Differentiate the following functions $W.r.t.$ 'x'.

   (i) $Sin(\log x), x > 0$

   (ii) $x^{\sin x}, x > 0.$

   (b) Find the equation of all lines having slop 2 and being tangent to the curve $y + \frac{2}{x-3} = 0.$

8. (a) If $z = log(x^3 + y^3 - x^2y - xy^2)$, prove that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = \frac{2}{x+y}$

(b) Optimize the function by using Lagrangian multipliers method :

$f(x, y) = x + y$, subject to constraint $x^2 + y^2 = 1$.

9. (a) Define linear, non-linear and Bernoulli differential equation with an example.

(b) Solve $(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$.

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