FORMULA BOOK

BY

Campusgate™

Number System

Classification of Numbers:

Natural Numbers (N) : 1, 2, 3, 4, ...
Whole Numbers (W): 0,1, 2, 3, 4, ...
Intezers (Z) = $-\infty$, ... –3, –2, –1, 0, 1, 2, 3, . . . ∞

Rational Numbers (Q): $\frac{p}{q}$ (where $q \neq 0$)

Irrational numbers (R – Q) = $x^{1/n} \neq Z$ and π, e

Rules related to Even and odd numbers

odd + odd = even	odd + even = odd
even + even = even	odd × odd = odd
odd × even = even	even × even = even
$\text{odd}^{\text{any number}} = \text{odd}$	$\text{even}^{\text{any number}} = \text{even}$

Converting recurring decimals into fractions

Model 1:
If all the digits after the decimal point are recurring, Say $x.\underbrace{\overline{abc..........}}_{n-\text{digits}}$

$$\Rightarrow x.\underbrace{\overline{abc..........}}_{n-\text{digits}} = x + \frac{abc......}{999....(n-\text{times})}$$

Model 2: If only some of the digits are recurring, Say $x.ab\overline{cde}$

$$\therefore x.ab\overline{cde} = x + \frac{abcde - ab}{99900}$$

Laws of Indices

1. $x^m \times x^n = x^{m+n}$
2. $x^m \div x^n = x^{m-n}$
3. $\left(x^m\right)^n = x^{mn}$
4. $x^0 = 1$
5. $\sqrt[m]{a^x} = a^{x/m}$
6. $\sqrt[mn]{a^x} = \sqrt[m]{a^{x/n}} = \sqrt[n]{a^{x/m}} = a^{x/mn}$
7. $\sqrt[n]{a^x} = \sqrt[m]{a^{mx/n}}$
8. $\sqrt[m]{a^{xy}} = \left(\sqrt[m]{a^x}\right)^y = \left(\sqrt[m]{a^y}\right)^x = \left(\sqrt[m]{a}\right)^{xy} = a^{\frac{xy}{m}}$

Divisibility Rules

Rule 1:
If $N = A \times B \times C....$ then the remainder when N is divided by D is equal to the product of the remainders when A, B, C ... are divided by D.

$$\Rightarrow \left(\frac{N}{D}\right)_R = \left(\frac{A}{D}\right)_R \times \left(\frac{B}{D}\right)_R \times \left(\frac{C}{D}\right)_R ...$$

Here $\left(\frac{N}{D}\right)_R$ means the remainder when N is divided by D

Rule 2:
If N = A + B + C.... then the remainder when N is divided by D is equal to the sum of the remainders when A, B, C ... are divided by D.

$$\Rightarrow \left(\frac{N}{D}\right)_R = \left(\frac{A}{D}\right)_R + \left(\frac{B}{D}\right)_R + \left(\frac{C}{D}\right)_R ...$$

General divisibility Rules

Let us a take a number ABCDEF. In decimal system this number can be written as
100,000A + 10,000B + 1000C + 100D + 10E + F

Divisibility Rule for 2:
We can easily observe that from rule 2, if ABCDEF has to be divisible by 2, 2 must divide all the six terms above. It is evident that except F remaining numbers are divisible by 2. So if F is divisible by 2 then the number ABCDEF is divisible by 2.

Divisibility Rule for 5:
Since all the terms except F is divisible by 5, the number is divisible when F is divisible by 5, or F must be 0 or 5.

Divisibility Rule for 4:
We can see that Except last two terms 10E and F, the remaining terms are divisible by 4. so If the last two digits are divisible by 4, the entire number is divisible by 4.

Divisibility Rule for 8:
Except last three terms the remaining terms are divisible by 8. So if the last three digits are divisible by 8 then the number is divisible by 8.

Thumb Rule: for 2, 4, 8, 16... we need to check the last 1, 2, 3, 4 ... digits. Observer there 1, 2, 3, 4 are the powers of the divisor with base 2.

Divisibility Rule for 3, 9:
100,000A + 10,000B + 1000C + 100D + 10E + F = 99999A + 9999B + 999C + 99D + 9E + (A + B + C + D + E + F)

We can see that Except (A + B + C + D + E + F) remaining terms are divisible by 3, 9. If the digit sum is divisible by 3, 9 respectively, then the number ABCDEF is divisible by 3, 9. (A + B + C + D + E + F) is called digit sum of a number.

Divisibility Rule for 11:
100,000A + 10,000B + 1000C + 100D + 10E + F = 100,001A + 9,999B + 1,001C + 99D + 11E + (-A + B - C + D - E + F)

From above we know that except (-A + B - C + D - E

+ F) the remaining digits are divisible by 11. So if the difference between the sum of the digits in the even places and odd places is 0 or multiple of 11 then the number is divisible by 11.

Divisibility Rule for 6, 12 or any composite number:
If a composite divisor can be written as a product of co-primes and each of these co-primes divide the given number exactly, then that number is divisible by the divisor. So if 2, 3 divide the given number exactly then 6 divides that number exactly. Similarly, divisibility for 12 is to check divisibility for 3, 4.

Divisibility Rule for 7, 13, 19
We use a special property to solve questions involving division with the above numbers.
If the divisor can divide a number in the format of 10^n+1 then we can group the given number into numbers of n and put positive and negative signs alternatively from the right hand side. For example, 13 divides 1001 which can be written as 10^3+1.
Similarly if the divisor can divide a number in the format of 10^n-1 then we can group the given numbers into numbers of n and put positive signs.

Divide the given number into several groups of 3 digits and put + and - signs alternatively from right hand side. Then find the sum and divide with 7, 13 to get remainders.

Fermat's little theorem

If P is a prime number then $\left(\dfrac{a^{p-1}}{p}\right)_R = 1$

Wilson's Theorem

If P is a prime number then (P - 1)! + 1 is divided by P the remainder is 0. $\left(\dfrac{(P-1)!+1}{P}\right)_R = 0$

(or) this can be written algebrically (p - 1)! + 1 = 0 (mod P).

Euler's Totient theorem

If a, N are coprimes with each other then, $\left(\dfrac{a^{\varphi(N)}}{N}\right)_R = 1$

Here $\varphi(N) = N\left(1-\dfrac{1}{a}\right)\left(1-\dfrac{1}{b}\right)...$ where N = $a^p \times b^q \times ...$

given, a, b ... are prime numbers.

Power Cycle

Power

Base		1	2	3	4	5
	2	2	4	8	6	2
	3	3	9	7	1	3
	7	7	9	3	1	7
	8	8	4	2	6	8
	4	4	6	4	6	4
	9	9	1	9	1	9

Formulas on Factors and Coprimes

Formula 1:

The number of factors of a number $N = a^p.b^q.c^r...$ is (p+1).(q+1).(r+1)...

Here $a^p.b^q.c^r...$ is the prime factorization of N.

Formula 2:

The sum of factors of a number $N = a^p.b^q.c^r...$

can be written as $\dfrac{a^{p+1}-1}{a-1}\times\dfrac{b^{q+1}-1}{b-1}\times\dfrac{c^{r+1}-1}{c-1}...$

Formula 3:
The number of ways of writing a number as a product of two number = $\dfrac{1}{2}\times[(p+1).(q+1).(r+1)...]$

(if the number is not a perfect square)

If the number is a perfect square then two conditions arise:

1. The number of ways of writing a number as a product of two distinct numbers =
$\dfrac{1}{2}\times[(p+1).(q+1).(r+1)...-1]$

2. The number of ways of writing a number as a product of two numbers and those numbers need not be distinct= $\dfrac{1}{2}\times[(p+1).(q+1).(r+1)...+1]$

Formula 4:
The number of co-primes of a number = N =
$a^p.b^q.c^r...$ $\phi(N) = N\times\left(1-\dfrac{1}{a}\right)\times\left(1-\dfrac{1}{b}\right)\times\left(1-\dfrac{1}{c}\right)...$

Formula 5:

The sum of co-primes of a number N= $\phi(N)\times\dfrac{N}{2}$

Formula 6:
The number of ways of writing a number N as a product of two co-prime numbers = 2^{n-1} where n=the number of prime factors of a number.

Formula 7:

Product of the factors of N = $N^{\left(\frac{\text{Number of factors}}{2}\right)}$

= $N^{\left(\frac{(p+1).(q+1).(r+1)....}{2}\right)}$

Rules on Counting Numbers

1. Sum of first n natural numbers = $\dfrac{n(n+1)}{2}$

2. Sum of first n odd numbers = n^2

3. Sum of first n even numbers = n(n + 1)

4. Sum of the squares of first n natural numbers = $\frac{n(n+1)(2n+1)}{6}$

5. Sum of the cubes of first n natural numbers = $\left[\frac{n(n+1)}{2}\right]^2$

H.C.F. & L.C.M. OF FRACTIONS

H.C.F of fractions = $\frac{\text{H.C.F of Numerators}}{\text{L.C.M. of Denominators}}$

L.C.M. of fractions = $\frac{\text{L.C.M of Numerators}}{\text{H.C.F. of Denominators}}$

Important formulas in HCF and LCM

LCM Formula 1:

The least number leaving remainder *r* in each case when divided by x, y, z = (LCM of x, y, z) + r

The series of such number will be (LCM of x, y, z) $\times$ n + r

Here n is a natural number.

LCM Formula 2:

The least number leaving remainders x_1, y_1, z_1 when divided by x, y , z where (x - x_1) = (y - y_1) = (z - z_1) = a is (LCM of x, y, z) - a. The series of such numbers will

(LCM of x, y, z) $\times$ n - a

HCF Formula 1:

If a, b, c are the remainders in each case when A, B, C are divided by N then N = HCF (A-a, B-b, C-c)

HCF Formula 2:

When A, B, C are divided by N then the remainder is same in each case then N = HCF of any two of (A-B, B-C, C-A)

Useful Formulas

1. $(a + b)^2 = a^2 + b^2 + 2ab$
2. $(a - b)^2 = a^2 + b^2 - 2ab$
3. $(a + b)^2 = (a - b)^2 + 4ab$
4. $a^2 - b^2 = (a - b)(a + b)$
5. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
6. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
7. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
8. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Percentages

$x\% = \frac{x}{100}$

Percentage conversions:

$\frac{1}{2} = 50\%$

$\frac{1}{3} = 33\frac{1}{3}\%$ or 33.33% ; $\frac{2}{3} = 66\frac{2}{3}\%$ or 66.66%

$\frac{1}{4} = 25\%; \frac{3}{4} = 75\%$

$\frac{1}{5} = 20\%; \frac{2}{5} = 40\%; \frac{3}{5} = 60\%; \frac{4}{8} = 80\%$

$\frac{1}{6} = 16\frac{2}{3}\%$ or 16.66%;

$\frac{1}{7} = 14\frac{2}{7}\%$ or 14.28% ; $\frac{2}{7} = 28\frac{4}{7}\%$ or 28.56%

$\frac{1}{8} = 12\frac{1}{2}\%$ or 12.5% ; $\frac{3}{8} = 37\frac{1}{2}\%$ or 37.5% ;

$\frac{5}{8} = 62\frac{1}{2}\%$ or 62.5% ;

$\frac{7}{8} = 87\frac{1}{2}\%$ or 87.5

$\frac{1}{9} = 11\frac{1}{9}\%$ or 11.11%

$\frac{1}{11} = 9\frac{1}{11}\%$ or 9.09%

Formula 1:

A is what percentage of B? $\Rightarrow \frac{A}{B} \times 100$

Formula 2:

A is how much percent greater than B? $\Rightarrow \frac{A-B}{B} \times 100$

Formula 3:

A is how much percent less than B? $\Rightarrow \frac{B-A}{B} \times 100$

Formula 4:

If A is increased by K% then the new number is

Method 1:

$(100+K)\% \times A = \frac{(100+K)}{100} \times A$

Method 2: Calculate K% of the given number and add to the original number A. $\Rightarrow A + (K\% \times A)$

Formula 5:

If A is decreased by K% then the new number is

Method 1:

$(100-K)\% \times A = \frac{(100-K)}{100} \times A$

Method 2:

Calculate K% of the given number and subtract from the original number A. $\Rightarrow A - (K\% \times A)$

Formula 6:

$$A\%(B) = B\%(A) = \frac{AB}{100}$$

Formula 7:
If several percentages are acting on the same number then we can add all the percentages.

$$x_1\%(K) + x_2\%(K) + .. = (x_1 + x_2 ...)\%(K)$$

Formula 8:
If a number K got increased by A% and B% successively then the final percentage is given by

$$\left(A + B + \frac{AB}{100}\right)\%$$

Note1: If decreased then substitute +A% with -A%
Note2: Any two dimensional diagram like square, rectangle, rhombus, triangle, circle, parallelogram, sides got increased or decreased by certain percentages, then the percentage change in the area can be calculated by the above formula.

Ratio and Proportion

If two ratios are equal then we say they are in proportion then

$a:b :: c:d \Rightarrow a \times d = b \times c$

or $\frac{a}{b} = \frac{c}{d} \Rightarrow a \times d = c \times d$

There exist four types of relations between 2 variables:
1. Direct Proportion
2. Inverse Proportion
3. Direct Relation
4. Inverse Relation

1. Direct Proportion:
If two variables x, y are directly proportional then $x \propto y$ or x = ky where k is a constant.

2. Inverse proportion:
If two variables x, y are inversely proportional then $x \propto \frac{1}{y}$ or xy = k where k is a constant.

3. Direct relation:
If two variables x, y are in direct relation then,
x = k + ny
Here K and n are some constants.

4. Indirect relation:
If two variables x, y are in indirect relation then,
x = k – ny

Partnership

1. Ratio of Division of Gains :
(i) When investments of all the partners are for the same time, the profit is distributed among the partners in the ratio of their investments.
Suppose A and B invest Rs. x and Rs. y respectively for a year in a business, then at the end of the year : (A's share of profit) : (B's share of profit) = x : y.
(ii) When investments are for different time periods, then equivalent capitals are calculated for a unit of time by taking (capital $\times$ number of units of time).
Suppose A invests Rs. x for 'p' months and B invests Rs. y for 'q' months, then (A's share of profit) : (B's share of profit) = xp : yq.

2. Working and Sleeping Partners :
A partner who manages the business is known as a **working partner** and the on who simply invests the money is a **sleeping partner.**

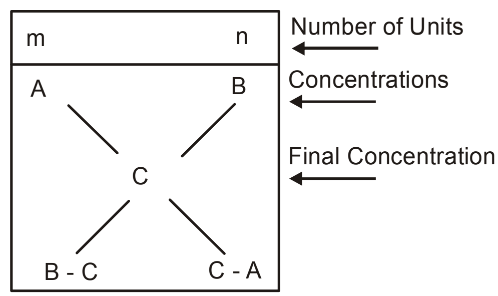
Averages

1. Average or Mean = $\frac{\text{Sum of observations}}{\text{Number of observations}}$

2. **Weighted Average:**
If the average of 'm ' quantities is 'A ' and the average age of ' n ' other quantities is 'B ' then the average of all of them put together is = C = $\frac{mA + nB}{m + n}$

Alligation Rule

Alligation rule helps us to find, in what ratio two mixtures with different concentrations are to be mixed to get a tar get concentration.



Alligation rule= $\frac{m}{n} = \frac{B - C}{C - A}$

Replacement Formula

The general formula for replacements is as follows:

$$FC = IC \times \left(1 - \frac{x}{V}\right)^n$$

Here
FC = Final concentration
IC = Initital concentration
x = replacement quantity
V = Final volume after replacement
n = number of replacements

Time and Work

1. If A can do a piece of work in x days, then in 1 day, A will do $\frac{1}{x}$ of the total work.
If 'A' worked k days at this rate, he could complete $\frac{k}{x}$ of the total work.

2. If A is thrice as good as B, then

(a) In a given amount of time, A will be able to do 3 times the work B does.
Ratio of work done by A and B (in the same time) = 3 : 1
(b) For the same amount of work, B will take thrice the time as A takes.
Ratio of time taken by A and B (same work done) = 1 : 3
3. Efficiency is directly proportional to the work done and inversely proportional to the time taken.

$$\left(\text{Efficiency} = \frac{\text{Total work}}{\text{days}}\right)$$

1. If 'A' can do a piece of work in x days and 'B' can do it in y days, then the fraction of work done by A and B together in 1 day = $\frac{1}{x}+\frac{1}{y}=\frac{x+y}{xy}$

or, time required to complete the work by both the men = $\frac{xy}{x+y}$

Pipes and Cisterns

This consists of problems on how long it will take for different pipes of different diameters to fill a cistern; the time taken to fill a cistern when one pipe is filling it while another empties it, etc.

Concepts

1. If a pipe can fill a tank in x hours and another pipe can fill it in y hours, then the fraction of tank filled by both pipes together in 1 hour = $\frac{1}{x}+\frac{1}{y}=\frac{x+y}{xy}$

or, time required to fill the tank by both the pipes = $\frac{xy}{x+y}$

2. If a pipe can fill a tank in x hours and another pipe can empty it in y hours, then the fraction of tank filled by both the pipes together in 1 hour = $\frac{1}{x}-\frac{1}{y}=\frac{y-x}{xy}$.

Profit loss and Discount

Cost Price: The rate at which a merchant buys goods. This is his investment
Selling Price: The rate at which a merchant sells his goods.
Marked Price: The rate at which a merchant rises his price above the cost price (may be anticipating some hagglers)

$$\begin{tikzcd} CP \arrow[r, "\text{Profit \%}", "\text{Loss \%}"'] \arrow[rr, bend right, "\text{Mark up \%}"'] & SP & MP \arrow[l, "\text{Discount \%}"'] \end{tikzcd}$$

Key Formulas:

Profit or Gain = Selling Price - Cost Price = SP - CP
Profit = $CP \times (\text{Profit\%})$

Loss = Cost price - Selling price = CP - SP
Loss = $CP \times (\text{Loss\%})$

Profit % = $\frac{SP-CP}{CP}\times 100 = \frac{\text{Profit}}{CP}\times 100$

Loss % = $\frac{CP-SP}{CP}\times 100 = \frac{\text{Loss}}{CP}\times 100$

Important: Profit or Loss always calculated on Cost price Only.

Discount = Marked price - Selling Price = MP - SP

Discount % = $\frac{MP-SP}{MP}\times 100 = \frac{\text{Discount}}{MP}\times 100$

Calculating Selling price from Cost price:

In the profit case selling price is greater than cost price, and this case we gain some profit. That is we are increasing the cost price by some percentage to get the selling price. This can be done in several ways

In profit case:

$CP + (\text{Profit\%}) \times CP = SP$

$(100 + \text{Profit})\% \times CP = SP$

$\Rightarrow \frac{(100+\text{profit})}{100}\times CP = SP$

In loss case:

$CP - CP \times \text{Loss\%} = SP$

$CP \times (100 - \text{Loss})\% = SP$

Calculating Selling Price from Marked Price:

$MP - MP \times \text{discount\%} = SP$

$MP \times \left(100 - \text{discount}\right)\% = SP$

Calculating Cost price from Selling Price:

This is the reverse operation of the above

In profit case: $\frac{SP}{(100+\text{Profit})\%} = CP$

Eg: A person sold a watch for Rs.1500 making 25% profit.

CP = $\frac{1500}{125\%} = 1500 \times \frac{100}{125}$

In loss Case: $\frac{SP}{(100-\text{Loss})\%} = CP$

Eg: A person sold a mobile for Rs.12000 making 20%

loss. CP = $\frac{12000}{80\%} = 12000 \times \frac{100}{80}$

A man sold two articles for Rs. P each. On selling first, he gains x% and on the other, he loses x%. This transaction always result in loss.

Loss % = $\frac{(\text{Common Gain and Loss})^2}{100} = \frac{x^2}{100}$

Two important Relations when Profit/Loss, Markup and Discount involved:

$$CP\left(\frac{100 \pm \text{Profit / Loss\%}}{100}\right) = MP\left(\frac{100 - \text{Discount\%}}{100}\right)$$

$$\left(\frac{100 \pm P/L\,\%}{100}\right) = \left(\frac{100 + \text{Markup\%}}{100}\right)\left(\frac{100 - \text{Discount\%}}{100}\right)$$

Time Speed Distance

1. Distance = Speed × Time

2. 1 km/hr = $\frac{5}{18}$ m/s

3. If the ratios of speed is a : b : c, then the ratios of time taken is : $\frac{1}{a}:\frac{1}{b}:\frac{1}{c}$, when distance is constant.

4. Average speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

Relative Speed:

1. If two objects are moving in 'opposite directions' towards each other at speeds u and v, then relative speed = Speed of first + Speed of second = u + v.

2. If the two objects move in the same direction with speeds u and v, then relative speed = difference of their speeds = u – v.

3. If the two objects start from A and B with speeds u and v respectively, and after crossing each other take a and b hours to reach B and A respectively, then u : v = $\sqrt{\frac{b}{a}}$

Tips for solving questions on trains

D = S X T

1. 1 Pole and I Train:
Length of The Train (m) = Speed of the Train (m/s) $\times$ Time taken to cross the pole (s)

2. 1 Train and 1 Platform
Length of the Train + Length of the Platform (m) = Speed of the Train (m/s) $\times$ Time taken to cross the platform (s)

3. 1 Train with speed speed V_1 and 1 moving person with speed V_2
Case 1: If both are moving in same direction
Length of The Train (m) = [Speed of the Train - Speed of the Man] (m/s) $\times$ Time taken to cross the man (s)

Case 2: If both are moving in opposite direction
Length of The Train (m) = [Speed of the Train + Speed of the Man] (m/s) $\times$ Time taken to cross the man (s)

4. 2 Trains with speeds V_1, V_2
Case 1: If both are moving in same direction
[Length of The Train 1 + Length of the Train 2](m) = [Speed of the Train1 - Speed of the Train 2] (m/s) $\times$ Time taken to cross (s)

Case 2: If both are moving in opposite direction
[Length of The Train 1 + Length of the Train 2](m) = [Speed of the Train1 + Speed of the Train 2] (m/s) $\times$ Time taken to cross (s)

Boats and Streams

Downstream motion of a boat is its motion in the same direction as the flow of the Stream.
Upstream motion is exactly the opposite.

There are two parameters in these problems.
1. Speed of the Stream (S): This is the speed with which the river flows.
2. Speed of the boat in still water (B): If the river is still, this is the speed at which the boat would be moving.
The effective speed of a boat in **upstream** = B – S
The effective speed of a boat in **downstream** = B + S

3. The speed of the boat in still water is given as B = $\frac{1}{2}(d+u)$, and the speed of the Stream S = $\frac{1}{2}(d-u)$, where d and u are the downstream and upstream speeds, respectively

Circular motion

The problems in circular motion deal with races on a circular track to calculate the time of meeting at the starting point and anywhere on the track.

1. If two people A and B start from the same point, at the same time and move in the same direction along a circular track and take x minutes and y minutes respectively to come back to the starting point, then they would meet for the first time at the starting point according to the formula given below:
First time meeting of A and B at the starting point = (LCM of x and y)
Note: This formula would remain the same even if

they move in the opposite directions.

2. If two people A and B start from the same point with speeds m km/hr and n km/hr respectively, at the same time and move in the same direction along a circular track, then the two would meet for the first time by the formula given below:
Time of the first meeting

$$= \frac{\text{Circumference of the track}}{\text{Relative speed}}.$$

Races

"Race" is a competition of speeds. All participants of a race are required to run a specific distance; whoever does it in the minimum will be the winner of the race.

When all participants reach the finishing point at the same instant of time, the race is said to end in a "Dead Heat"

The various types of phrases used in problems on races and their interpretations are as follows:

1. **A gives B y meters:** This means, both A and B start at the starting point at the same instant of time, but while A reaches the finishing point, B is y meters behind. This indicates that A is the winner of the race.

2. **A gives B t minutes:** This means, both A and B start at the starting point at the same instant, but B takes t minutes more as compared to A to finish the race. Here also, A is the winner.

3. **A can give B a start of y meters:** A starts from the starting point and B starts y meters ahead, but still both A and B reach the finishing point at the same instant of time. So, the race ends in a dead heat.

4. **A can give B a start of t minutes:** A starts t minutes after B starts from the starting point, but still, both A and B reach the finishing point at the same instant of time. So, again the race ends in a dead heat.

5. **A gives B y meters and t minutes:** A and B start at the starting point at the same instant, but while A reaches the finishing point, B is behind by y meters, and, B takes t minutes compared to A to complete the race. So, B covers remaining y meters y meters in extra t minutes. This gives the speed of B as y/t.

Simple Interest and Compound Interest

Simple Interest = SI = $\frac{P \times T \times R}{100}$

Amount in Compound Interest = A = $P\left(1+\frac{R}{100}\right)^n$

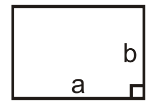
Compound Interest = CI = A – P

$= P\left(1+\frac{R}{100}\right)^n - P$

Areas

Rectangle:

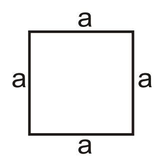
In rectangle opposite pairs of sides are parellel and equal. All four sides are perpendicular to each other. Long side and short side lengths are different. 4 right angles exist.



Area = ab
Perimeter = 2(a + b)

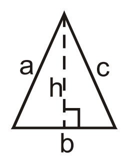
Square:

In square all 4 sides are equal. Opposide sides are parellel to each other. 4 right angles exist.



Area = a^2
Perimeter = 4a
Diagnol = $\sqrt{2}a$

Triangle:

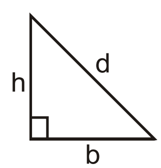


Area = $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}bh$

Area = $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

Rightangle Triangle:

A pair of sides are perpendicular to each other.

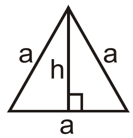


Area = $\frac{1}{2}bh$

Hypotenuse = d = $\sqrt{h^2 + b^2}$

Equilateral Triangle:

All the three sides are equal. Angle between any two sides is equal to 60.



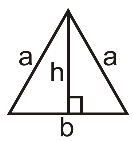
Area = $\frac{\sqrt{3}}{4}a^2$

Height = $\frac{\sqrt{3}}{2}a$

Perimeter = 3a

Isosceles Triangle:

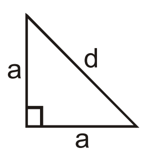
Two sides are equal.



Area = $\frac{b}{4}\sqrt{4a^2 - b^2}$

Isosceles Right angle triangle:

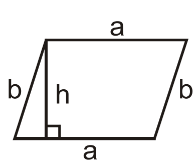
Two sides are equal and these two sides are perpendicular to each other.



Area = $\frac{1}{2}a^2$

Parellogram:

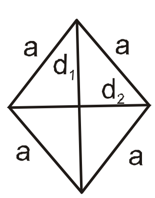
Two opposite pairs of sides are parellel. But there is no perpendicularity.



Area = base $\times$ height = $a \times h$

Rhombus:

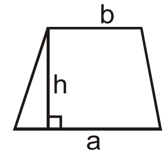
All 4 sides are equal and diagnols are perpendicular to each other. Sides are not perpendicular to each other. Perpendiculars bisect each other.



Area = $\frac{1}{2} \times d_1 \times d_2$

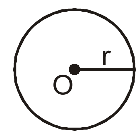
Trapezium:

Only a pair of lines are parellel to each other. Other two sides are not parellel.



Area = $\frac{1}{2} \times h \times (a + b)$

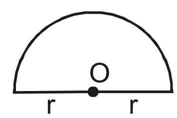
Circle:



Area = πr^2 here $\pi \approx 3.1416$ or $\pi \approx \frac{22}{7}$

Perimeter = $2\pi r$

Semi Circle:

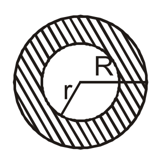


Area = $\frac{\pi r^2}{2}$

Perimeter = $\pi r + 2r$

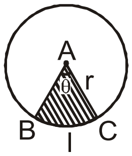
Ring:

Two circles which are concentric forms ring.
Bigger circle radius = R, and smaller circle radius = r



Area = $\pi R^2 - \pi r^2$

Sector:



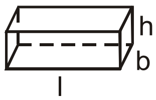
Area = $\frac{\theta}{360^0} \times \pi r^2$

Perimeter = $\frac{\theta}{360^0} \times 2\pi r + 2r$

Volumes

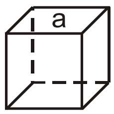
Cuboid:

All rectangular rooms look like cuboids.



Volume = lbh
Total surface area = 2(lb + bh + lh)
Lateral surface area = 2 (bh + lh)

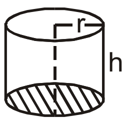
Cube:



Volume = a^3

Total surface area = $6a^2$

Cylinder:

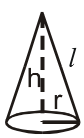


Volume = $\pi r^2 h$

Total surface area = $2\pi rh + 2\pi r^2$

Lateral surface area = $2\pi rh$

Cone:

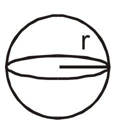


Volume = $\frac{1}{3}\pi r^2 h$

Lateral surface area = $\pi rl = \pi r\sqrt{h^2 + r^2}$

Total surface area = $\pi r^2 + \pi r\sqrt{h^2 + r^2}$

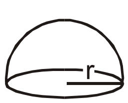
Sphere:



Volume = $\frac{4}{3}\pi r^3$

Total surface area = $4\pi r^2$

Hemi Sphere:



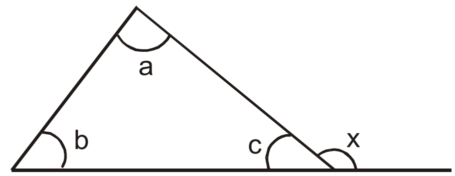
Volume = $\frac{2}{3}\pi r^3$

Lateral Surface area = $2\pi r^2$

Total Surface area = $3\pi r^2$

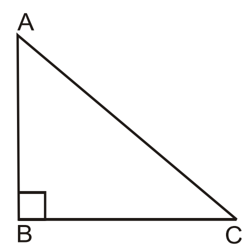
Plane Geometry

Triangle and Angle:



1. The sum of three angles of a triangle is 180°.
   $\angle a + \angle b + \angle c = 180°$.
2. The exterior angle of a triangle is equal to the sum of the two interior opposite angles.
   $\angle x = \angle a + \angle b$

   Pythagoras' theorem: In a right angle triangle the square of the hypotenuse is equal to the sum of the squares of other two sides.

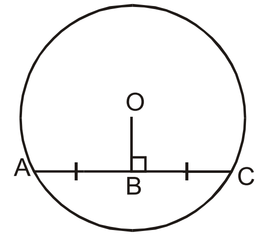


$AC^2 = AB^2 + BC^2$

Pythagorean triplet: There are certain triplets which satisfy the pythagoras' theorem and are commonly, called pythagorean triplet.
For example: 3, 4, 5; 5, 12, 13; 24, 10, 26; 24, 7, 25; 15, 8, 17

Circles

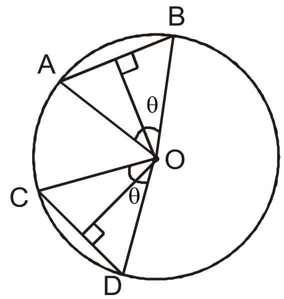
1. If two circles have equal radii, then both circles are congruent.
2. The perpendicular drawn from the centre of a circle to a chord bisects the chord.



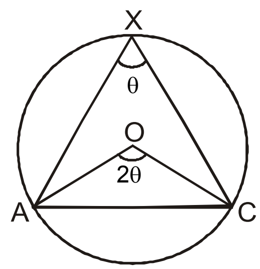
3. Equal chords of a circle are equidistant from the

centre.

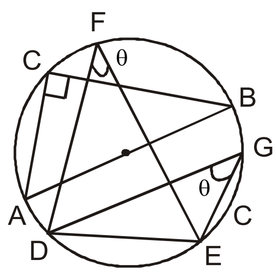
4. Equal chords of a circle subtend equal angles of the centre.



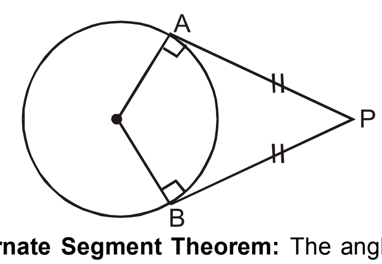
5. The angle subtended by an arc of a circle at the centre is double the angle subtended by it on the circle in the other segment.



6. Angles subtended by a chord in the same segment are equal. In the below figure, DE makes equal angles at F and G.
7. The angle in semicircle is 90°. AB makes 90° at C.

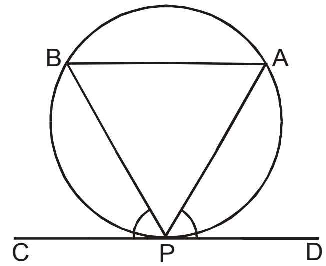


8. Tangent is perpendicular to radius.
9. The length of the two tangents that can be drawn from an external point to the same circle, are equal. PA = PB

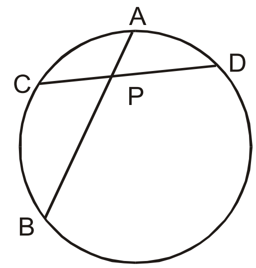


10. **Alternate Segment Theorem:** The angle made by a chord and tangent to the circle is equal to the angle made by the chord in the alternate segment. This is called the alternate segment theorem.

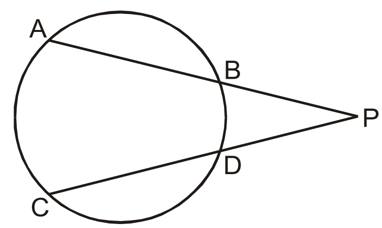
$\angle BPC = \angle BAP$ and $\angle APD = \angle ABP$



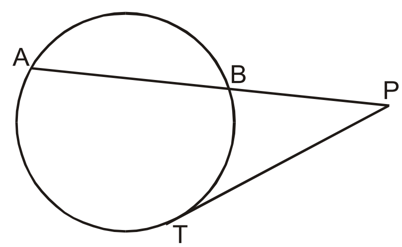
11. If two chords AB and CD intersect each other internally, then
PA × PB = PC × PD



12. If two line segments (extended chords) PA and PC intersect each other externally, then
PA × PB = PC × PD



13. If PBA is a secant and PT is a tangent segment, then
$PA \times PB = PT^2$



14. Sum of all the interior angles of a polygon = (n – 2) × 180°
where n is the number of sides in the polygon.

Coordinate Geometry

Formula 1.

Distance between the points (x_1, y_1) and (x_2, y_2) is

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Formula 2.

The area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3)

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Formula 3:
The coordinate of the mid-point of the line joining the points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Line joining vertices to the mid points of the opposite sides of a triangle are called medians.
Point of intersection of medians is called 'centroid.

Formula 4:
Centroid of a triangle. The centroid of a triangle whose vertices are $(x_1\ y_1)$, $(x_2\ y_2)$ and (x_3, y_3) is

$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Point of intersection of perpendicular bisectors of sides of triangle is called as 'circumcenter'. Point of intersection of altitudes of triangle is called 'ortho-center'.

Formula 5:
Angle made by the line with the positive direction of x - axis is called the inclination of the line.
If θ is the inclination, then 'tan θ' denotes the slope of the line.
Slope of the line joining the points (x_1, y_1) and (x_2, y_2) is $\frac{y_2-y_1}{x_2-x_1}$; $(x_1 \neq x_2)$. The slope is also indicated by m.

Formula 6:
If the slopes of two lines be m_1 and m_2, then the lines will be
(i) parallel if $m_1 = m_2$ (ii) perpendicular if $m_1m_2 = -1$
Area of quadrilateral formed by points (x_1, y_1) (x_2, y_2) (x_3, y_3) and (x_4, y_4) taken in order is

$$= \frac{1}{2}\begin{vmatrix} x_1 - x_3 & x_2 - x_4 \\ y_1 - y_3 & y_2 - y_4 \end{vmatrix}$$

Formula 7:
In $\triangle ABC$ if AD is the median drawn to BC, then $AB^2 + AC^2 = 2(AD^2 + CD^2)$

Formula 8:
Slope Intercept form: All straight lines can be written as $y = mx + c$,
where m is the slope of the straight line, c is the Y intercept or the Y coordinate of the point at which the straight line cuts the Y-axis.

Formula 9:
Point slope form. The equation of a straight line passing through (x_1, y_1) and having a slope m is $y - y_1 = m(x - x_1)$.

Formula 10:
Two point form. The equation of a straight line passing through two points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Formula 11:
Intercept form: The intercept form of a line is

$$\frac{x}{a} + \frac{y}{b} = 1$$

Where 'a' is the intercept on x-axis and 'b' is the intercept on y-axis.
The point of intersection of any two lines of the form $y = ax + b$ and
$y = cx + d$ is same as the solution arrived at when these two equations are solved.

Formula 12:

The length of perpendicular from a given point (x_1, y_1) to a given line $ax + by + c = 0$ is $\left|\frac{ax_1 + by_1 + c}{\sqrt{(a^2+b^2)}}\right| = p$,

where p is the length of perpendicular. In particular, the length of perpendicular from origin (0, 0) to the line $ax + by + c = 0$ is $\frac{c}{\sqrt{a^2+b^2}}$

Formula 13:
Distance between two parallel straight lines $ax + by + c_1 = 0$ abd $ax + by + c_2 = 0$ is given by

$$\frac{|c_1 - c_2|}{\sqrt{a^2+b^2}}$$

Formula 14:
If m_1 and m_2 are slopes of two straight lines, then acute angle (θ) between them is given by

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

Consistency of Equations

When a system of equations has at least one solution, we say that the system is consistent. When it has no solution we say that the system is inconsistent. Let the system of equation be

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

$$\Rightarrow \frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

and $(a_1 b_2 \neq a_2 b_1)$

(i) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the equations are consistent with unique solution.

(ii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the equations are consistent with infinite solutions.

(iii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the equations are inconsistent i.e, no solution.

Arithmetic progression (Key points):

A series in which each term (except first term) differs from its preceding term by a fixed quantity is called an Arithmetic progression (A.P.) and the fixed quantity is called the common difference. If a is the first term and d is the common difference of an A.P. then that A.P is a + (a+d) + (a+2d)+..... . If the same quantity is added (multiplied) to each term of an A.P. then the resulting series is also an A.P.

1. n th term of an A.P: T_n = a+ (n -1) d.

2. Sum to first n terms of an A.P. = S_n = $\frac{n}{2}\left[2a+(n-1)d\right]$

3. If we know the last term of a series then = $\frac{n}{2}[a+l]$ where l is the last term.

Arithmetic Mean:

1. If a,x,b are in A.P. then x is called Arithmetic mean (A.M) between a and b. The arithmetic mean between a and b is $\frac{a+b}{2}$.

2. If three numbers are in A.P., then they can be taken as a – d, a, a+d.

3. If four numbers are in A.P., then they can be taken as a – 3d, a – d, a + d, a + 3d.

4. If five numbers are in A.P., then they can be taken as a – 2d, a – d, a, a + d, a + 2d.

Geometric progression (Key points)

A series in which each term (except first term) is obtained by multiplying the preceding term by a fixed quantity is called a Geometric progression (G.P) and the fixed quantity is called the common ratio. If a is the first term and r is the common ratio of a G.P. then that G.P is $a+ar+ar^2+......$. If every term of a G.P. is multiplied by a fixed real number, then the resulting series is also a G.P. If every term of a G.P is raised to the same power, then the resulting series is also a G.P. The reciprocals of the terms of a G.P. is also a G.P.

1. nth term of a G.P : ar^{n-1}

2. The sum of first n terms of a G.P = $\frac{a(r^n-1)}{r-1}$ if r>1;

= $\frac{a(1-r^n)}{1-r}$ if r<1

3. The sum of infinite G.P. = $\frac{a}{1-r}$ when |r| <1

Geometric Mean:

1. If three positive numbers a,x,b are in G.P. then x is called the Geometric Mean (G.M) between a and b. The G.M. between two positive numbers a and b is $\sqrt{a.b}$

2. If three numbers are in G.P., then they can be taken $\frac{a}{r}$, a, ar.

3. If four numbers are in G.P., then they can be taken as $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

4. If five number are in G.P., then they can be taken as $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

Harmonic progression (Key points)

If the reciprocals of the terms of a series form An A.P., then the series is called a Harmonic progression (H.P.). If a,x,b are in H.P., then x is called Harmonic Mean between a and b.

1. The harmonic mean between two non zero numbers a and b is $\frac{2ab}{a+b}$.

2. If A, G, H are the arithmetic mean, geometric mean, harmonic mean between two positive numbers then $A \geq G \geq H$ and $G^2 = AH$

3. The sum of first n natural numbers = $\sum n = \frac{n(n+1)}{2}$

4. The sum of first n natural numbers = $\sum n^2 = \frac{n(n+1)(2n+1)}{6}$

5. The sum of cubes of first n natural numbers = $\sum n^3 = \frac{n^2(n+1)^2}{4}$

6. The sum of first n even integers = n(n + 1).

7. The sum of first n odd integers = n^2.

Arithmetico Geometric progression (Key points):

A series in which each term is the product of two factors so that the first factor is a term of an A.P and the second factor is a corresponding term of a G.P is called Arithmetic Geometric series.

In an AGP series a + (a + d)r + (a + 2d) r^2 + ...

nth term = $\{a + (n - 1)d\}\ r^{n-1}$.

1. Sum of first n terms =

$$\frac{a}{1-r}+\frac{dr(1-r^{n-1})}{(1-r)^2}-\frac{\{a+(n-1)d\}r^n}{(1-r)}$$

2. Sum of infinite terms = $\frac{a}{1-r}+\frac{dr}{(1-r)^2}$ if $|r|<1$

Permutations and Combinations

Note: Factorial of zero is 1 (0! = 1)

Formula 1:
"r" distinct objects can be arranged in "r" places in r! ways.

Formula 2:
Out of "r" objects "a" objects are similar, "b" objects are similar . . . the these "r" objects can be arranged in $\frac{r!}{a!\times b!\times ...}$ ways.

Formula 3:
"r" objects can be selected out of "n" objects in ^nC_r ways.

Formula 4:
"r" objects can be selected out of "n" objects and arranged in $= {}^nC_r \times r! = {}^nP_r$ ways.

Formula 5:
If a function can be done in x ways and for each of these functions the other function can be done in y ways then ***both*** the functions can be done in x × y ways.

Formula 6:
If a function can be done in x ways and the other function can be done in y ways then ***either of the functions*** can be done in x + y ways.

Formula 7:
. Division of m+n+p objects into three groups is given by $\frac{(m+n+p)!}{m!\times n!\times p!}$

If any of m,n or p equal to each other we have to divide by that factorial.

Formula 8:
Sum of all the n digit numbers that can be formed using all of the digits **with out repetition** = (n – 1)! × (111..n times) × (Sum of the digits)

Formula 9:
Sum of all the n digit numbers that can be formed using all of the digits **with repetition** = n^{n-1} × (111..n times) × (Sum of the digits)

Formula 10:
Sum of all the n digit numbers that can be formed using all of the digits **with out repetition and zero is one the the number** = [(n – 1)! × (111..n times) × (Sum of the digits)] – [(n – 2)! × (111..(n-1) times) × (Sum of the digits)]

Formula 11:
Sum of all the n digit numbers that can be formed using all of the digits **with repetition and zero is one the the number** = [n^{n-1} × (111..n times) × (Sum of the digits)] – [n^{n-2} × (111..(n-2) times) × (Sum of the digits)]

Formula 12:
Number of selections out of 'n' articles where atleast one article can be chosen = $2^n - 1$

Formula 13:
If n similar articles are to be distributed to r persons, $x_1+x_2+x_3+.....+x_n=n$ each person is eligible to take any number of articles then the total ways are = $^{(n+r-1)}C_{(r-1)}$

Formula 14:

If n similar articles are to be distributed to r persons, $x_1+x_2+x_3+.....+x_n=n$ each person must get minimum one article then the total ways are = $^{n-1}C_{r-1}$

Formula 15:
If n persons are seated around a circular table then they can be arranged in (n – 1)! ways.

Formula 16:
When two dice are rolled, any number from 2 to 7 happens for (n – 1) ways. Also, any number from 8 to 12 happens for (13 – n) ways.

Formula 17:
When 3 dice are rolled, any number from

3 to 8 can happen for $^{n-1}C_2$

13 to 18 can happen for $^{20-n}C_2$

9 happen for 25 times
10 happen for 27 times
11 happen for 27 times
12 happen for 25 times