Seminar 6

Practise Session

Before Going in…

- If I do chapter 6, we will definitely not finish it today.
- Therefore, rather going into a new topic, let’s do some recap!
- I have prepared some examples from chapters 1 to 5. Some will be easy, but some can be hard.
- Good Luck!

Basics in Quantum Mechanics

- Remember the Kronecker Product?

$$A \otimes B \equiv \left.\overbrace{\begin{bmatrix} A_{11}B & A_{12}B & \dots & A_{1n}B \\ A_{21}B & A_{22}B & \dots & A_{2n}B \\ \vdots & \vdots & \vdots & \vdots \\ A_{m1}B & A_{m2}B & \dots & A_{mn}B \end{bmatrix}}^{nq}\right\} mp\,.$$

- 1. Let $|\psi\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$. Write out $|\psi\rangle^{\otimes 2}$ and $|\psi\rangle^{\otimes 3}$ explicitly.
- 2. How can we write the Hadamard Transform in one qubit? Use the bra-ket notation.
- 3. What will $|\psi\rangle^{\otimes n}$ look like?

Incoherent States

- 1. Consider a pure ensemble of identically prepared spin ½ systems. Suppose we know the expectation values for Sx and Sz, and the sign of Sy are known. Determine the state vector.
- 2. Consider a mixed ensemble of spin ½ systems. The ensemble averages are all known. How can we construct the 2-by-2 density matrix?
- 3. Give an expression for the time evolution of the density operator.
- 4. In an ensemble of a spin-1 system, how many independent parameters do we need? What must we know except [Sx], [Sy] and [Sz]?

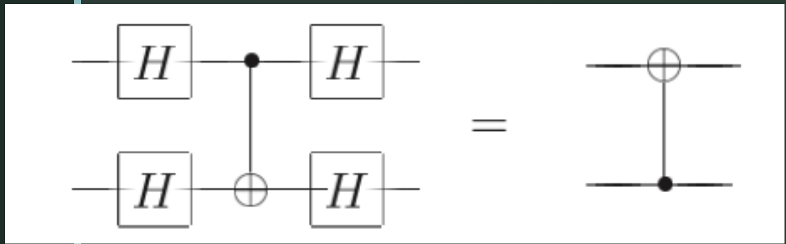
Universality of NAND

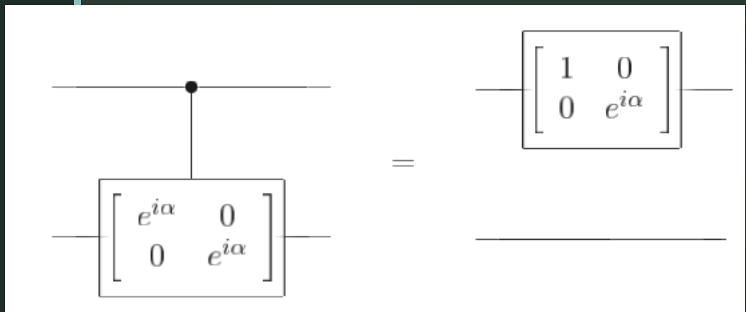
- 1. Construct the NOT gate.
- 2. Construct the AND gate.
- 3. Construct the OR gate.
- 3. Construct the NOR gate. (Inverted OR)
- 4. Construct the XOR gate.

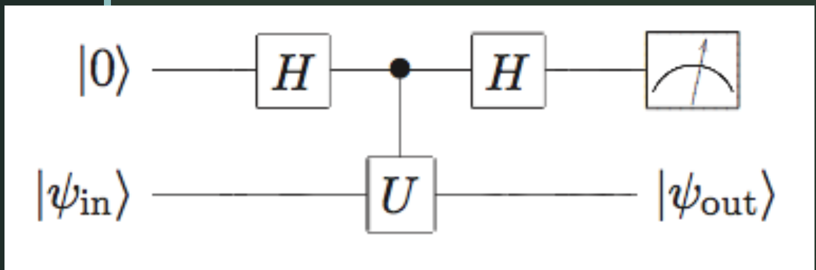
Single-Qubit Operations

- Remember the Pauli matrices and the rotation operators? If not, just look at seminar 3.
- 1. Show that $XYX=-Y$. What does this mean?
- 2. Prove $\exp(iAx) = \cos(x)I + i\sin(x)A$.

Controlled Gates







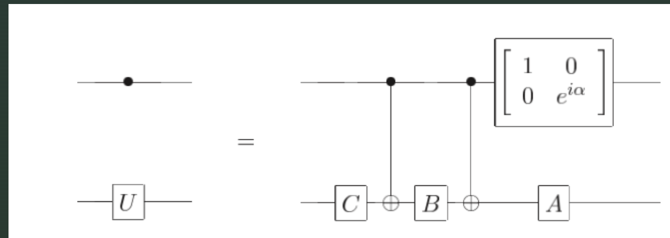
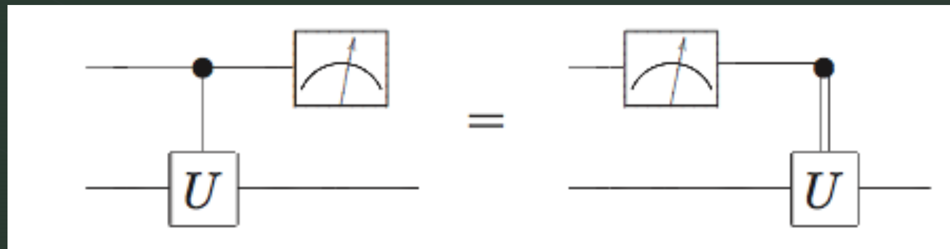


Figure 4.6. Circuit implementing the controlled-U operation for single qubit U. α, A, B and C satisfy $U = \exp(i\alpha)AXBXC$, $ABC = I$.



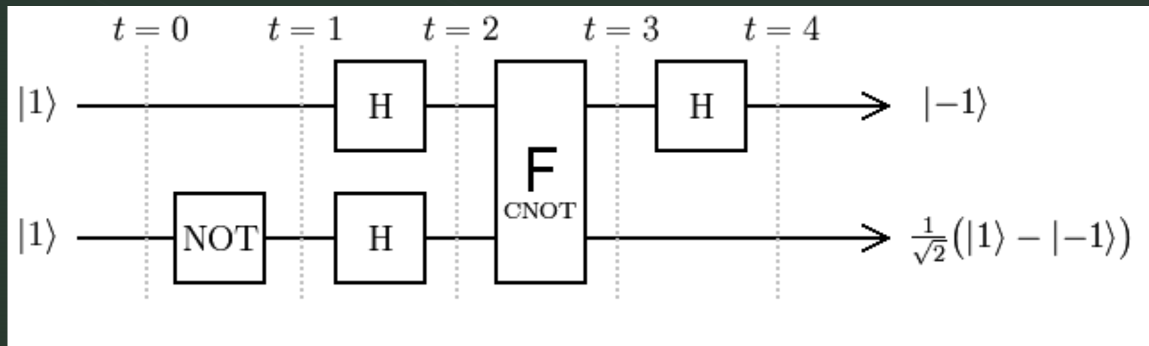
Double lines indicate classical bit.

Universality

- 1. Decompose the following to a two-level gate.

$$\frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}$$

- 2. Verify
-



Fourier Transform

- 1. Fourier tranfrom $|00\ldots0\rangle$.
- 2. Give a decomposition of the controlled-Rk gate into single qubit and CNOT gates.
- 3. Construct the circuit for the Inverse Fourier Transform.

Factoring

- 1. Remember U $|y\rangle \equiv |xy(\text{mod } N)\rangle$? Prove this is unitary.
- 2. Prove $\frac{1}{\sqrt{r}}\sum_{s=0}^{r-1}|u_s\rangle = |1\rangle$. I'll give you some hints if required.
- 3. Factor 91 using Shor's algorithm. It won't be the best way you've seen :D
- 4. Recently, scientists factored 15 using NMR. Why 15?