1. $(X, Y) \sim f(x, y)$.

$$f(x,y)=\begin{cases}1, & |y|<x,\quad 0<x<1;\\ 0, & 其他\end{cases}$$

求条件密度 $f(x|y)$.

2. $(X, Y) \sim f(x, y)$

$$f(x,y)=\begin{cases}3x, & 0<x<1,\quad 0<y<x\\ 0, & 其他\end{cases}$$

(1) $f_X(x)$, $f_Y(y)$　　(2) X, Y 是否独立.

3. $Y \sim Exp(\lambda)$, 定义随机变量 $X_k=\begin{cases}0, & Y\le k\\ 1, & Y>k\end{cases}$, $k=1,2$.

求 X_1, X_2 的联合分布列

4. $X \sim N(\mu, 4^2)$, $Y \sim N(\mu, 5^2)$. 比较 P_1, P_2 大小:

$P_1 = P(X \le \mu - 4)$, $P_2 = P(Y \ge \mu + 5)$

5. $X \sim P(\lambda)$. 如 $P(X=1)=P(X=2)$, 求 $P(X=4)$

6. X, Y 同分布, $X \sim f_X(x)=\begin{cases}\frac{3}{8}x^2, & 0<x<2\\ 0, & 其他\end{cases}$

已知 $A=\{X>a\}$ 与 $B=\{Y>a\}$ 独立, 且 $P(A\cup B)=\frac{3}{4}$

求 a.

1. $X \sim N(1.7, 3)$.

$$F_Y(y) = P(Y \le y) = P\left(X \ge \frac{1-y}{2}\right) = 1 - P\left(X < \frac{1-y}{2}\right) = 1 - F_X\left(\frac{1-y}{2}\right)$$

$$f_Y(y) = f_X\left(\frac{1-y}{2}\right) \cdot \frac{1}{2}$$

$$= \frac{1}{2\sqrt{6\pi}} \cdot e^{-\frac{\left(\frac{1-y}{2} - 1.7\right)^2}{6}}$$

2. $E[|X-Y|] = \int_0^1 \int_0^1 |x-y| \, dx\, dy = \int_0^1 \left(\frac{1}{2}x^2 + \frac{1}{2}(1-x^2)\right) dx = \frac{1}{3}$

~~$Var(|X-Y|) =$~~

$$E[|X-Y|^2] = E[X^2] - 2E[XY] + E[Y^2] = \cancel{1}\ \tfrac{2}{3} - 2E[XY]$$

$$E[XY] = \int_0^1 \int_0^1 xy \, dx\, dy = \int_0^1 x\, dx \int_0^1 y\, dy = \frac{1}{4}$$

$$\Rightarrow E[|X-Y|^2] = \frac{1}{6}$$

$$Var(|X-Y|) = \frac{1}{6} - \frac{1}{9} = \frac{1}{18}$$

3. (1) ~~$P(X=1) =$~~

$$P\left(\frac{X-b}{a-b} = 1\right) = P(X=a) = p$$

$$P\left(\frac{X-b}{a-b} = 0\right) = P(X=b) = 1-p.$$

(2) $Var(X) = E(X^2) - E(X)^2$

$$E(X^2) = a^2 p + b^2(1-p)$$

$$E(X) = ap + b(1-p)$$

$$Var(X) = a^2 p + b^2(1-p) - a^2 p^2 - b^2(1-p)^2 - 2abp(1-p)$$

$$= a^2 p(1-p) + b^2(1-p)p - 2abp(1-p)$$

$$= (a-b)^2 p(1-p)$$

4.

I_A	0	1
P	~~0.8413~~ 0.6826	~~0.1587~~ 0.3174
	$1-p$	p

I_B	0	1
P	0.9545	0.0455
	$1-q$	q

$E[I_A]=0.3174$

$E[I_B]=0.0455$

$E[I_A^2]=0.3174$, $E[I_B^2]=0.0455$

$\Rightarrow Var(I_A)=0.21665724$

$Var(I_B)=0.04342975$

$Var(I_A+I_B)$

$E[(I_A+I_B)^2]=4pq+(1-p)q+p(1-q)=2pq+p+q=0.3917834.$

5. $E[X]=\int_0^1 x\cdot(ax+bx^2)\,dx=\frac{a}{3}+\frac{b}{4}=0.6$

$\int_0^1(ax+bx^2)\,dx=\frac{a}{2}+\frac{b}{3}=1$

$\Rightarrow \frac{b}{12}=-0.2 \Rightarrow b=-2.4,\ a=3.6$

$E[X^2]=\int_0^1 x^2(ax+bx^2)\,dx=\frac{a}{4}+\frac{b}{5}=0.9-\frac{12}{25}=\frac{45}{50}-\frac{24}{50}=\frac{21}{50}$

$Var(X)=E[X^2]-E[X]^2=\frac{21}{50}-\frac{9}{25}=\frac{3}{50}$

6. $f(x)=2xe^{-x^2}.\ x>0.$

$E[X]=\int_0^{+\infty}x\cdot 2xe^{-x^2}\,dx=\int_0^{+\infty}x\,d(-e^{-x^2})$

$=-xe^{-x^2}\Big|_0^{+\infty}+\int_0^{+\infty}e^{-x^2}\,dx$

$=\frac{\sqrt{\pi}}{2}$

$E[X^2]=\int_0^{+\infty}x^2\cdot 2xe^{-x^2}\,dx=-x^2e^{-x^2}\Big|_0^{+\infty}+\int_0^{+\infty}e^{-x^2}\cdot d\,x^2$

$=-e^{-x^2}\Big|_0^{+\infty}=1$

$\Rightarrow Var(X)=1-\frac{\pi}{4}$