Applications and Replications

VAR Toolbox 4.0

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Applications and replications

- Six canonical applications from the monetary macroeconomics literature, one for each identification strategy
  - Stock and Watson (2001) ⟶ Zero short-run restrictions
  - Blanchard and Quah (1989) ⟶ Zero long-run restrictions
  - Uhlig (2005) ⟶ Sign restrictions
  - Antolín-Díaz and Rubio-Ramírez (2018) ⟶ Narrative sign restrictions
  - Gertler and Karadi (2015) ⟶ External instruments
  - Jordà and Taylor (2025) ⟶ Local projections (LP-OLS and LP-IV)

- All scripts are in `Replic/` and named `GO_<Author><Year>.m`

Applications and Replications

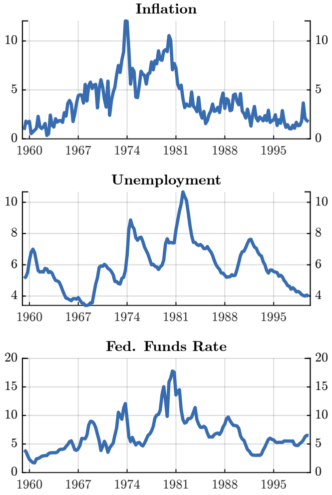
Stock and Watson (2001, JEP)

Stock and Watson (2001): Zero short-run restrictions

- Stock and Watson (2001). "Vector Autoregressions," *Journal of Economic Perspectives*

- US quarterly data from 1960:Q1 to 2000:Q4

- Three variables: inflation (π_t), unemployment (u_t), federal funds rate (r_t)



What are the effects of monetary policy on output?

- **Objective** Infer the causal influence of monetary policy (MP) on unemployment and inflation
- VAR with $p = 4$ and a constant: inflation (π_t), unemployment (u_t), federal funds rate (r_t)
- **Key identifying assumptions**
  - MP (r_t) responds contemporaneously to inflation and unemployment
  - MP shocks do not affect inflation or unemployment within the quarter

$$\begin{bmatrix} \pi_t \\ u_t \\ r_t \end{bmatrix} = \sum_{p=1}^{4} \Phi_p x_{t-p} + \begin{bmatrix} b_{11} & 0 & 0 \\ b_{21} & b_{22} & 0 \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \\ \varepsilon_t^{MonPol} \end{bmatrix}$$

Replicating Stock and Watson (2001) with the VAR Toolbox

- Load the data, set the lag order, and configure recursive (`'short'`) identification with bootstrap inference

```
% Data and lag order
X = [DATA.infl, DATA.unemp, DATA.ff];
nlags = 4;  detc = 1;
% Identification setup and impulse responses with bootstrap bands
VARopt = VARoption;
VARopt.vnames    = {'Inflation','Unemployment','Fed Funds'};
VARopt.nsteps    = 24;
VARopt.snames    = {bfeps('1'), bfeps('2'), bfeps('MonPol')};
VARopt.ident     = 'short';
VARopt.inference = 1;
```

Replicating Stock and Watson (2001) with the VAR Toolbox

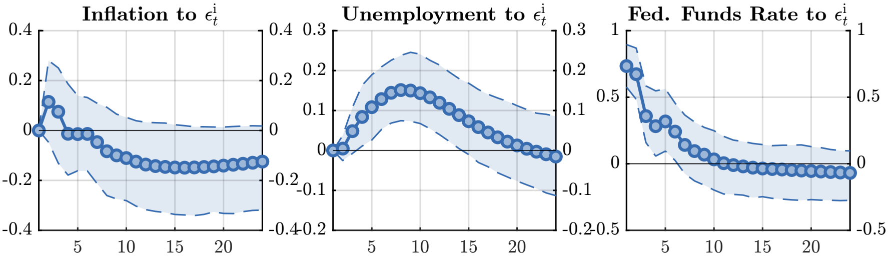
- Estimate the VAR and plot the impulse responses with confidence bands

```
VAR = VARmodel(X, nlags, detc, VARopt);
VARirplot(VAR.IRbar, VARopt, VAR.IRinf, VAR.IRsup);
```

- **Note:** the **ordering of the variables matters** — inflation is first, unemployment second, policy rate third; monetary policy shocks cannot affect prices or output contemporaneously

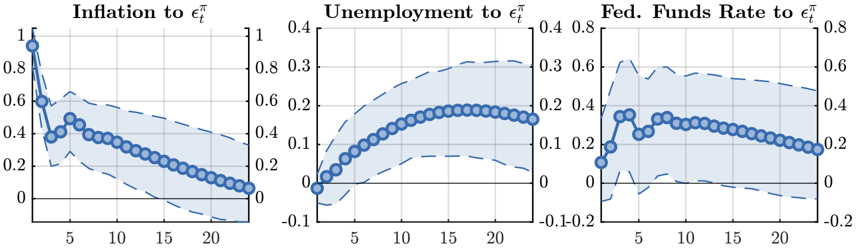
The effect of a monetary policy shock

- Contractionary MP shock raises inflation in the short run (price puzzle) and unemployment



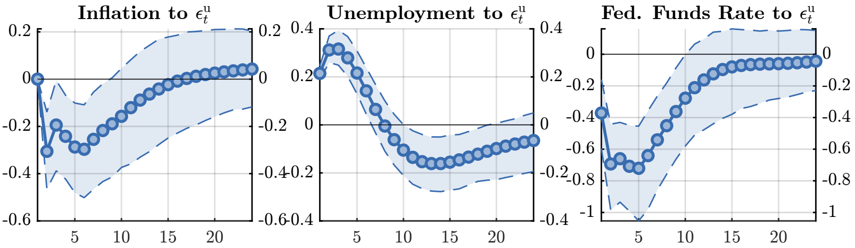
Shock ε_t^1: Supply-like pattern

- Shock ε_t^1 (ordered first) affects all three variables contemporaneously and behaves as a negative aggregate supply shock

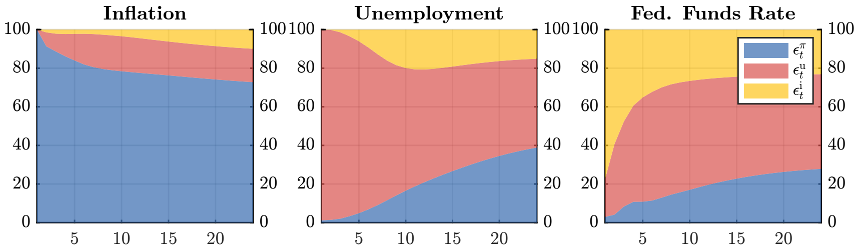


Shock ε_t^2: Demand-like pattern

- Shock ε_t^2 (ordered second) affects unemployment and the policy rate contemporaneously but not inflation, behaving as a negative aggregate demand shock



Forecast error variance decomposition and historical decomposition

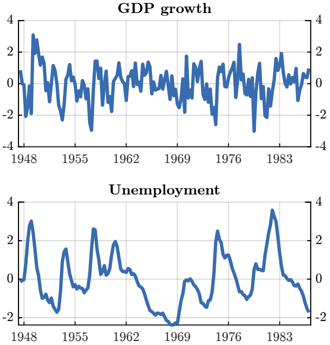


Applications and Replications

Blanchard and Quah (1989, AER)

Blanchard and Quah (1989): Zero long-run restrictions

- Blanchard and Quah (1989). "The Dynamic Effects of Aggregate Demand and Supply Disturbances," *American Economic Review*
- US quarterly data from 1948:Q1 to 1987:Q4
- Two variables: GNP growth (Δy_t), unemployment (u_t)



What are the effects of demand and supply shocks?

- **Objective** Identify the effects of demand and supply shocks on output and unemployment

- Bivariate VAR with $p = 8$: output growth (Δy_t) and unemployment (u_t)

- **Key identifying assumption** Demand shocks have no long-run effect on the output *level*; supply shocks do

- The long-run impact matrix $C \equiv (I - \Phi)^{-1}B$ must be lower triangular:

$$\begin{bmatrix} \Delta y_{t,t+\infty} \\ u_{t,t+\infty} \end{bmatrix} = \begin{bmatrix} c_{11} & 0 \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_t^{Supply} \\ \varepsilon_t^{Demand} \end{bmatrix}$$

Replicating Blanchard and Quah (1989) with the VAR Toolbox

- Load the data, set the lag order, and configure long-run (`'long'`) identification with bootstrap inference

```
% Data and lag order
X = [DATA.y, DATA.u]; % GNP growth, unemployment
nlags = 8;  detc = 1;
% Identification setup and impulse responses with bootstrap bands
VARopt = VARoption;
VARopt.vnames    = {'GNP growth','Unemployment'};
VARopt.nsteps    = 40;
VARopt.frequency = 'q';
VARopt.snames    = {bfeps('Supply'), bfeps('Demand')};
VARopt.ident     = 'long';
VARopt.inference = 1;
```

Replicating Blanchard and Quah (1989) with the VAR Toolbox

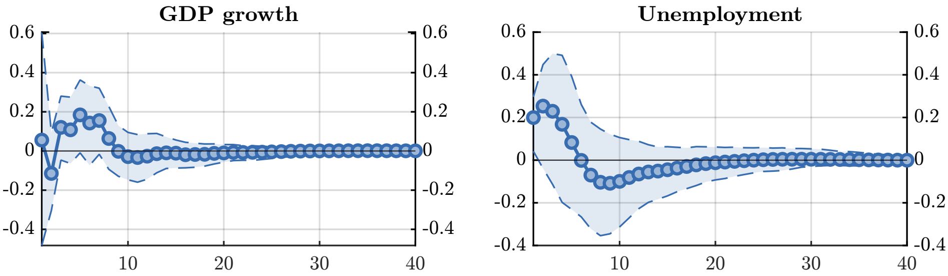
- Estimate the VAR and plot the impulse responses

```
VAR = VARmodel(X, nlags, detc, VARopt);
VARirplot(VAR.IRbar, VARopt, VAR.IRinf, VAR.IRsup);
```

- **Note:** output growth is ordered first; the zero in the upper-right of C imposes long-run neutrality of demand shocks on output

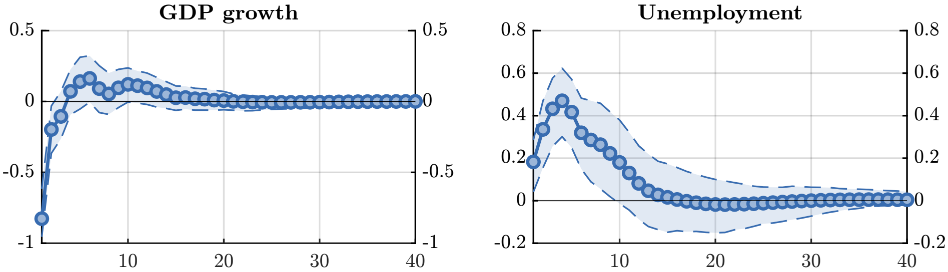
Aggregate supply shock

- Supply shock permanently raises output



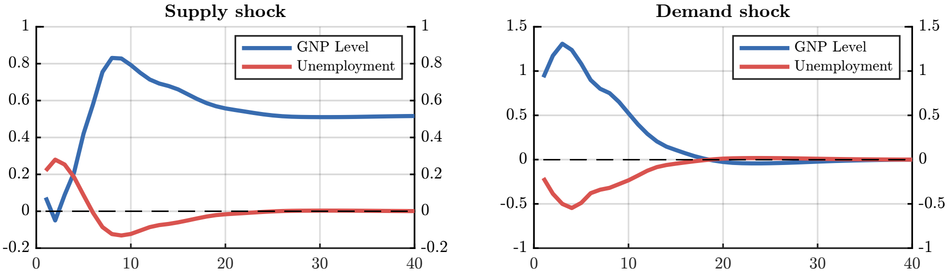
Aggregate demand shock

- Demand shock has a hump-shaped effect on output and unemployment, with no long-run effect on output level — as imposed by construction



Long-run output level responses (Figures 1 and 2 from BQ)

- Cumulative sum of output growth responses (output level); demand shock converges to zero by construction

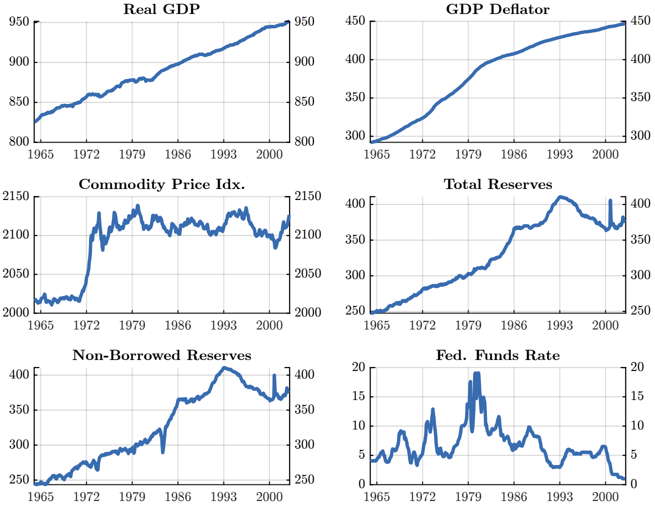


Applications and Replications

Uhlig (2005, JME)

Uhlig (2005): Sign restrictions

- Uhlig (2005). "What are the effects of monetary policy on output?" *Journal of Monetary Economics*
- US monthly data from 1965:M1 to 2003:M12



What are the effects of monetary policy on output?

- **Objective** Infer the causal effect of monetary policy on real GDP with minimal identifying restrictions
- Six-variable VAR with $p = 12$: real GDP, GDP deflator, commodity prices, total reserves, non-borrowed reserves, federal funds rate
- **Key restrictions** A contractionary MP shock should (for $h = 0, \ldots, 5$)
  - Raise the federal funds rate
  - Lower prices and non-borrowed reserves
  - Leave real GDP unrestricted (that is the object of interest!)

Replicating Uhlig (2005) with the VAR Toolbox

- Set up the six-variable VAR, the number of rotation draws, and the sign-restriction horizon (`sr_hor`)

```
% Data and lag order
X = [DATA.y, DATA.pi, DATA.comm, DATA.res, DATA.nbres, DATA.ff];
nlags = 12;  detc = 1;
% Identification setup and impulse responses
VARopt = VARoption;
VARopt.ndraws    = 500;
VARopt.sr_hor    = 6;
VARopt.inference = 1;
VARopt.nsteps    = 60;
VARopt.frequency = 'm';
VARopt.pctg      = 68;
VARopt.snames    = {bfeps('MonPol')};
```

Replicating Uhlig (2005) with the VAR Toolbox

- Encode the sign restrictions in matrix R (column 1 is the MP shock; rows are variables) and select `'sign'` identification

```
% R: 1 = positive, -1 = negative, 0 = unrestricted
R = [ 0, 0, 0, 0, 0, 0;    % Real GDP (unrestricted)
     -1, 0, 0, 0, 0, 0;    % GDP Deflator (negative)
     -1, 0, 0, 0, 0, 0;    % Commodity Prices (negative)
      0, 0, 0, 0, 0, 0;    % Total Reserves (unrestricted)
     -1, 0, 0, 0, 0, 0;    % NonBorr. Reserves (negative)
      1, 0, 0, 0, 0, 0];  % Fed Funds (positive)
% Identify with sign restrictions
VARopt.ident = 'sign';
VARopt.R = R;
```

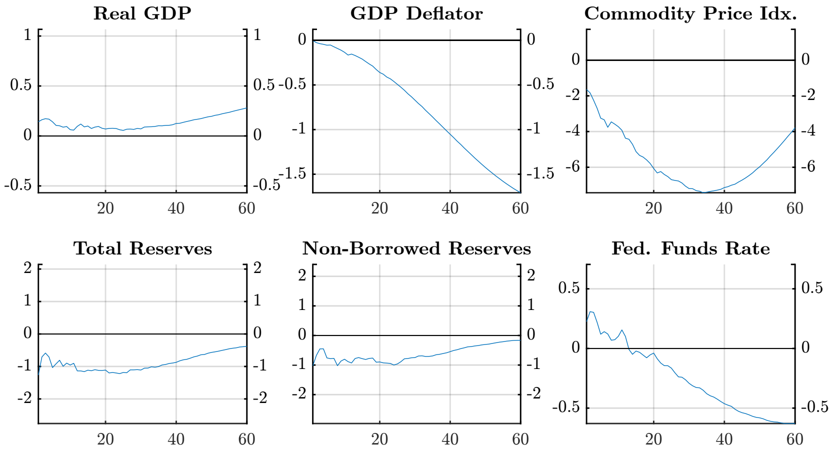
Replicating Uhlig (2005) with the VAR Toolbox

- Estimate and plot; for set-identified (sign-restricted) schemes the central tendency is the median (`IRmed`)

```
VAR_sr = VARmodel(X, nlags, detc, VARopt);
VARirplot(VAR_sr.IRmed, VARopt, VAR_sr.IRinf, VAR_sr.IRsup);
```

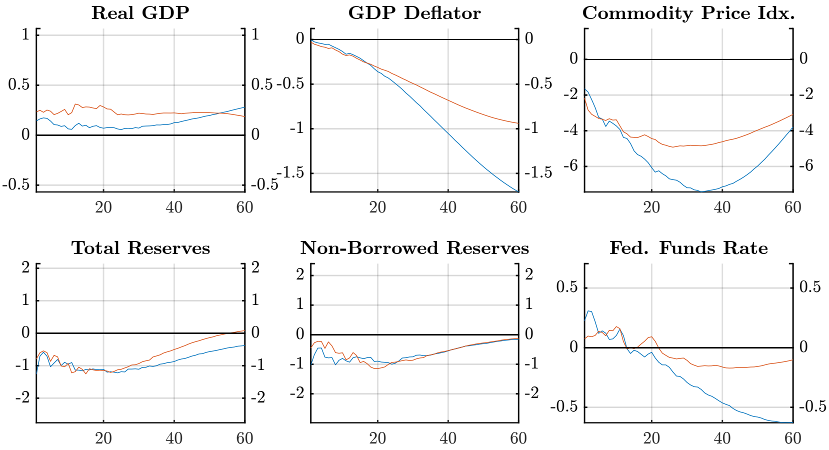
Sign restriction search: First accepted rotation

- Draw random orthonormal matrices Q until one $\tilde{B} = PQ$ satisfies restrictions...



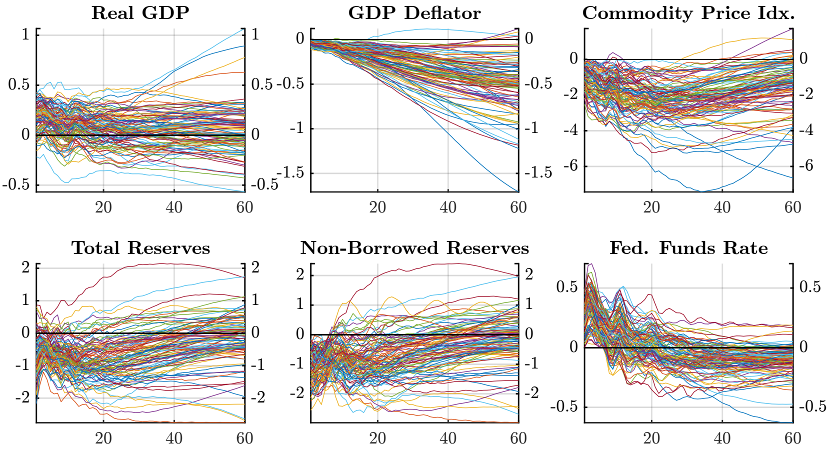
Sign restriction search: Two accepted rotations

- Draw another Q, check restrictions again... the second accepted $\tilde{B}$ is a different candidate



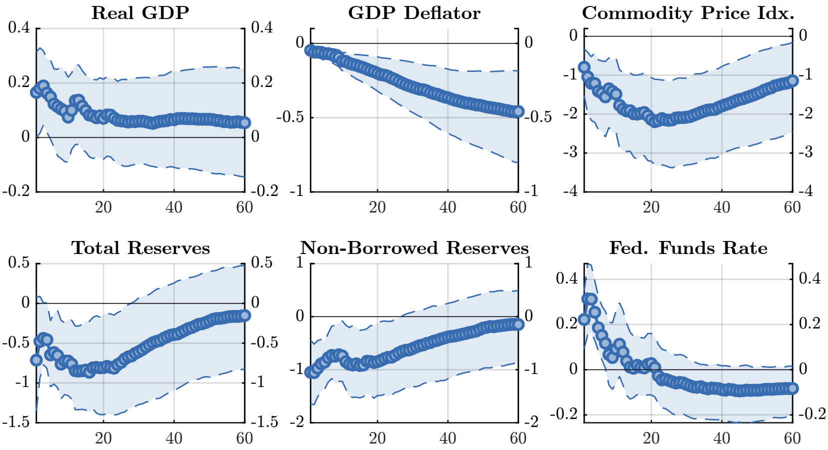
Sign restriction search: After many draws

- After N accepted rotations, the swathe of impulse responses spans the identified set



What are the effects of monetary policy on output?

- Output response is ambiguous $\Longrightarrow$ long-run monetary neutrality is not rejected

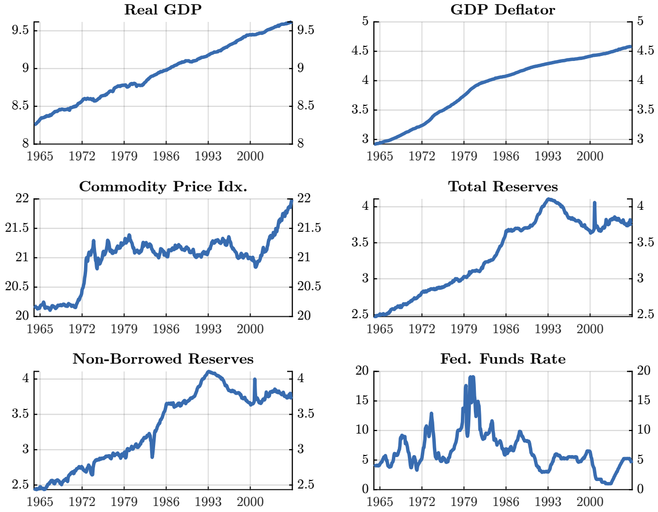


Applications and Replications

Antolín-Díaz and Rubio-Ramírez (2018, AER)

ADRR (2018): Narrative sign restrictions

- Antolín-Díaz and Rubio-Ramírez (2018). “Narrative Sign Restrictions for SVARs,” *American Economic Review*
- Same six-variable VAR as Uhlig (2005)
- Extended sample: 1965:M1 to 2007:M11; estimated without a constant



Narrative episode: October 1979

- The Volcker disinflation announcement provides two pieces of narrative evidence about 1979:M10

- **Type 1 (sign):** Historical records unambiguously place the monetary policy shock in positive territory that month

$$\varepsilon_{1979:M10}^{MonPol} > 0$$

- **Type 2 (dominance):** No comparably large competing shock was operating simultaneously $\Rightarrow$ monetary policy was the dominant driver of the unexpected movement in the federal funds rate

$$\left| b_{6m}\, \varepsilon_{1979:M10}^{MonPol} \right| > \sum_{k \neq m} \left| b_{6k}\, \varepsilon_{k,1979:M10} \right|$$

Replicating ADRR (2018) with the VAR Toolbox

- Set up the VAR (no constant: `detc = 0`, data demeaned) and the sign-restriction matrix `R.sign`

```
% Data and lag order
X = [DATA.y, DATA.pi, DATA.comm, DATA.res, DATA.nbres, DATA.ff];
nlags = 12;  detc = 0;
% Identification setup and impulse responses
VARopt = VARoption;
VARopt.nsteps = 60;
VARopt.sr_hor = 6;
VARopt.snames = {bfeps('MonPol')};
VARopt.dates  = dates;
% Sign restrictions matrix (column 1: MP shock)
R.sign = [ 0, 0, 0, 0, 0, 0;
          -1, 0, 0, 0, 0, 0;
          -1, 0, 0, 0, 0, 0;
           0, 0, 0, 0, 0, 0;
          -1, 0, 0, 0, 0, 0;
           1, 0, 0, 0, 0, 0];
```

Replicating ADRR (2018) with the VAR Toolbox

- Add the two narrative restrictions targeted at October 1979 (the Volcker reform)

```
% Type 1: MP shock positive at October 1979
R.narr_sign.shock  = 1;
R.narr_sign.period = '1979m10';
R.narr_sign.sign   = 1;
% Type 2: MP shock dominant for ff at October 1979
R.narr_dom.shock  = 1;
R.narr_dom.period = '1979m10';
R.narr_dom.var    = 6;
```

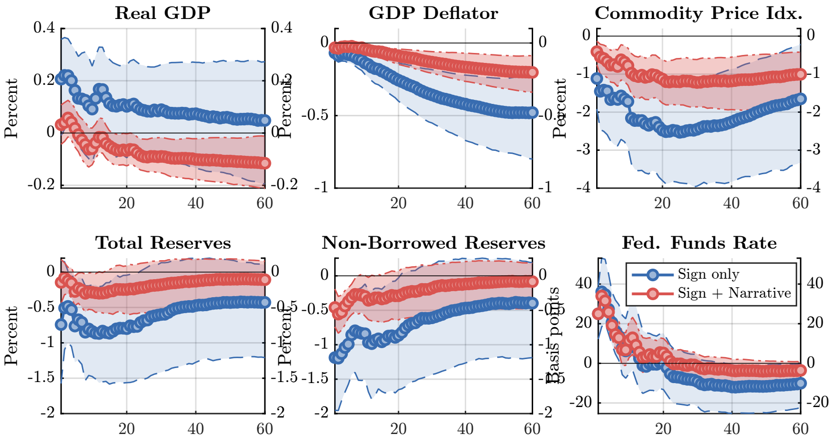
Replicating ADRR (2018) with the VAR Toolbox

- Combine sign and narrative restrictions, estimate, and plot

```
% Identify with sign + narrative restrictions
VARopt.ident = 'sign';
VARopt.R = R;
VAR_nsr = VARmodel(X, nlags, detc, VARopt);
VARirplot(VAR_nsr.IRmed, VARopt, VAR_nsr.IRinf, VAR_nsr.IRsup);
```

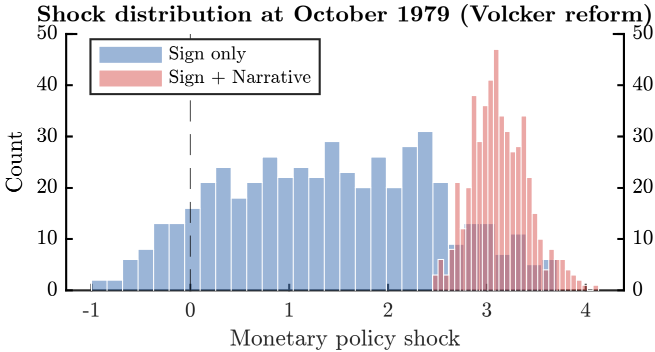
Narrative restrictions tighten the identified set

- Adding the October 1979 restrictions substantially narrows credible bands and shifts the central estimate (red) relative to sign restrictions alone (blue)



Distribution of the MP shock at the narrative episode

- Distribution of $\varepsilon^{MonPol}_{1979:M10}$ across accepted draws. Sign restrictions alone (blue) leave a wide distribution; the dominance restriction (red) concentrates mass on large positive values



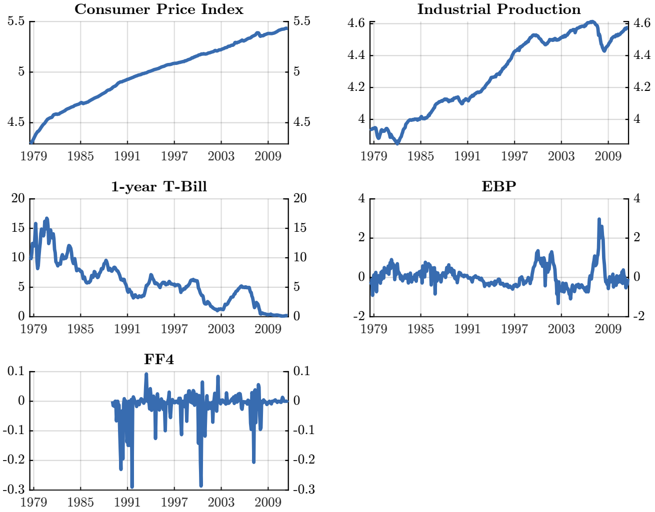
Applications and Replications

Gertler and Karadi (2015, AEJ:M)

Gertler and Karadi (2015): External instruments

- Gertler and Karadi (2015). "Monetary Policy Surprises, Credit Costs, and Economic Activity," *American Economic Journal: Macroeconomics*

- US monthly data from 1979:M7 to 2012:M6



What are the effects of monetary policy on credit markets?

- **Objective** Identify the credit channel of monetary transmission
- Four-variable VAR with $p = 12$: 1-year T-bill rate, CPI, industrial production, excess bond premium (EBP)
- **Key identifying element** Instrument z_t constructed from high-frequency movements in federal funds futures prices around FOMC announcements:

$$z_t = -(P_{t,\tau+20} - P_{t,\tau-10})$$

- **1-year T-bill rate ordered first** (required by the IV implementation, which instruments the first equation's residuals)

Replicating Gertler and Karadi (2015) with the VAR Toolbox

- Load the data and the FF4 futures-surprise instrument; select the wild bootstrap for heteroskedasticity-robust bands

```
% Data, instrument, and lag order
X  = [DATA.gs1, DATA.logcpi, DATA.logip, DATA.ebp];
IV = DATA.ff4_tc;  % FF4 futures surprise
nlags = 12;  detc = 1;
% Options
VARopt = VARoption;
VARopt.nsteps    = 48;
VARopt.frequency = 'm';
VARopt.method    = 'wild';
VARopt.ndraws    = 500;
VARopt.inference = 1;
VARopt.pick      = 1;
```

Replicating Gertler and Karadi (2015) with the VAR Toolbox

- Identify the same VAR two ways — recursive (Cholesky) ordering and external instrument (proxy SVAR) — to compare them

```
% (1) Recursive (Cholesky) identification: policy rate ordered first
VARopt.ident      = 'short';
VAR_chol = VARmodel(X, nlags, detc, VARopt);
% (2) External-instrument identification (proxy SVAR)
VARopt.ident      = 'iv';
VARopt.IV         = IV;
VAR_iv   = VARmodel(X, nlags, detc, VARopt);
```

Instrument relevance: First-stage F-statistic

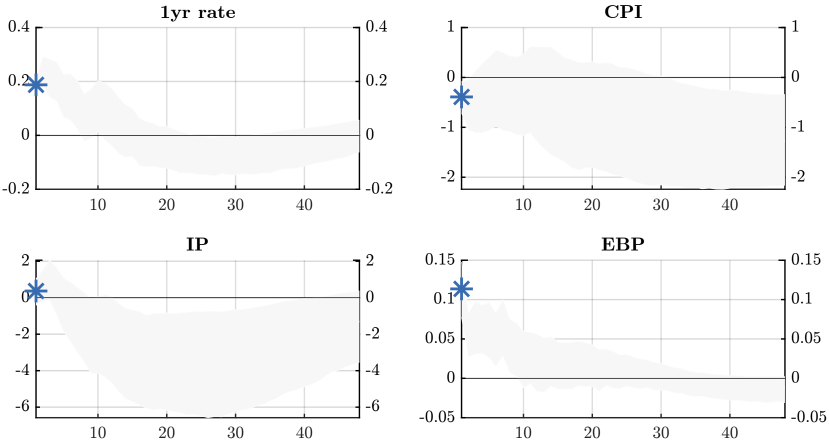
- A weak instrument biases IV estimates regardless of exogeneity. Rule of thumb Stock and Yogo (2002): $F > 10$
- The first-stage regression results are stored in `VAR_iv.FirstStage`; the F-statistic is at `VAR_iv.FirstStage.F`

```
% Full first-stage output and the F-statistic
disp(VAR_iv.FirstStage)
F_stat = VAR_iv.FirstStage.F;
```

- **Key fields:** `.F` (F-statistic), `.beta` (first-stage slope $\hat{\alpha}$), `.tstat`, `.rsqr` (first-stage R^2)

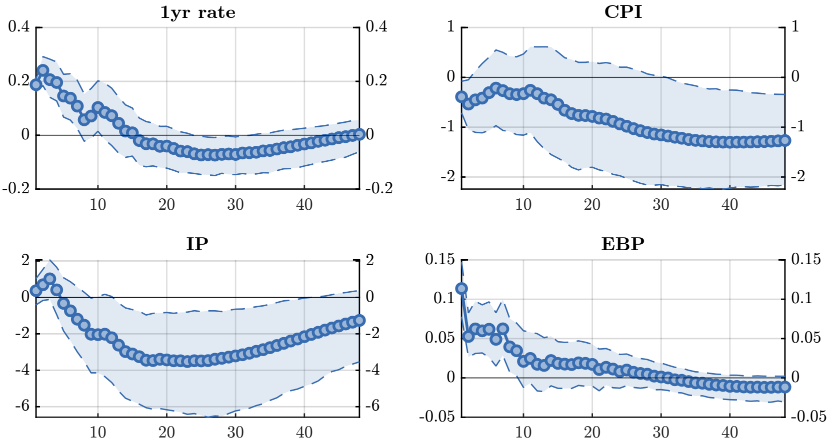
Impulse response functions: Impact effect

- The impact effect (b column) is recovered via the first- and second-stage IV regressions



Impulse response functions: Dynamic effect

- Contractionary MP shock raises 1-year rate and EBP, lowers industrial production and CPI



Applications and Replications

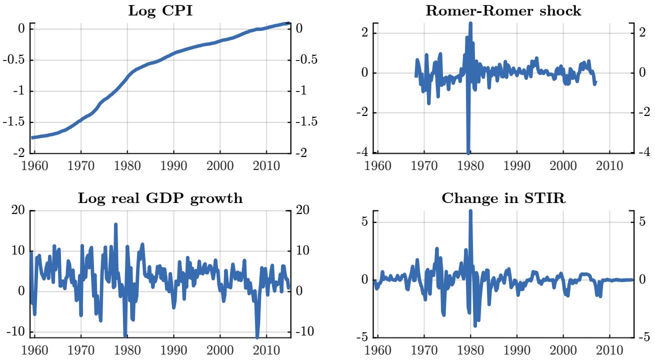
Jordà and Taylor (2025)

Jordà and Taylor (2025): Local projections

- Jordà and Taylor (2025). "Local Projections," *Journal of Economic Literature*
- **Objective** Illustrate LP-OLS and LP-IV on two canonical monetary applications
    - **Example 5 (LP-OLS):** Response of log CPI to a Romer-Romer narrative shock. Quarterly US data from 1985:Q1 to 2007:Q4. Horizon: 18 quarters
    - **Example 6 (LP-IV):** Response of unemployment to a federal funds rate shock, instrumented by the Romer-Romer (RRCG) narrative shock. Monthly US data from 1985:M1 to 2000:M1. Horizon: 49 months

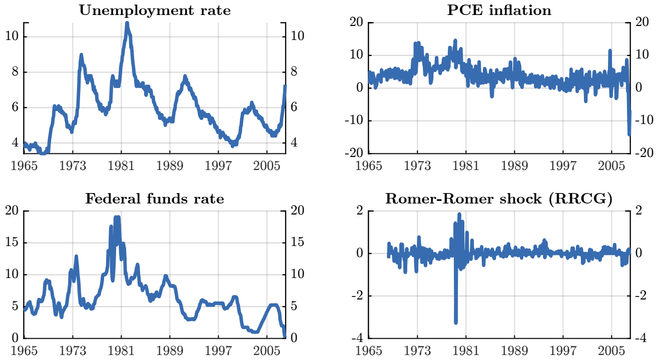
Example 5 (LP-OLS): Data

- Quarterly US data from 1985:Q1 to 2007:Q4
- Variables: log CPI ($\times 100$), Romer-Romer shock, log GDP growth, change in short-term interest rate
- CPI growth rate excluded (outcome is CPI level in long-difference form)



Example 6 (LP-IV): Data

- Monthly US data from 1985:M1 to 2000:M1
- Variables: unemployment rate (outcome), inflation, federal funds rate (endogenous treatment), RRCG narrative shock (instrument)
- Outcome: long-difference $\text{urate}_{t+h} - \text{urate}_{t-1}$



Identification: LP-OLS vs. LP-IV

- **LP-OLS** The Romer-Romer narrative shock s_t is assumed exogenous and enters the regression directly as the treatment. No instrumentation is required
- **LP-IV** The federal funds rate d_t is endogenous (the Fed responds to economic conditions). The RRCG narrative shock z_t instruments d_t via 2SLS at each horizon
- **Both use long-difference LHS:**
$$y^*_{t+h} \equiv y_{t+h} - y_{t-1}$$
Appropriate for persistent series; avoids level-stationarity issues

Replication: LP-OLS (Example 5)

- CPI response to the Romer-Romer shock; LHS in long-difference form; 18-quarter horizon. `LPmodel` returns a struct `LP_OLS` with fields `.IR`, `.INF`, `.SUP`

```
% LP-OLS -- log CPI response to Romer-Romer shock (quarterly)
LPopt_OLS          = LPoption;
LPopt_OLS.nsteps   = 18;          % 18-quarter horizon
LPopt_OLS.longdiff = 1;           % LHS: lcpi_{t+h} - lcpi_{t-1}
LPopt_OLS.impact   = 1;           % unit shock
LPopt_OLS.pctg     = 95;
CTRL_OLS = [dlrgdp dlcpi dstir];
LP_OLS    = LPmodel(lcpi, rr_shock, CTRL_OLS, 4, 1, LPopt_OLS);
IR_OLS      = LP_OLS.IR;
INF95_OLS   = LP_OLS.INF;
SUP95_OLS   = LP_OLS.SUP;
```

- `LPmodel` signature: `LPmodel(Y, S, CTRL, nlag, const, LPopt)` — `S` is the shock or instrument; lag order `nlag` applies to controls

Replication: LP-IV (Example 6)

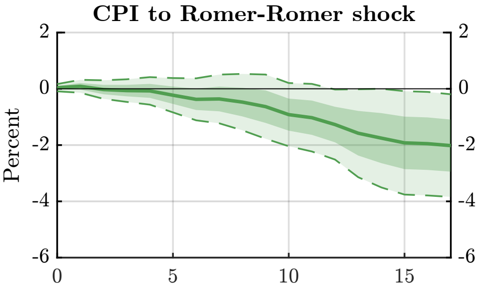
- Unemployment response to federal funds rate shock; RRCG narrative shock as instrument; 49-month horizon

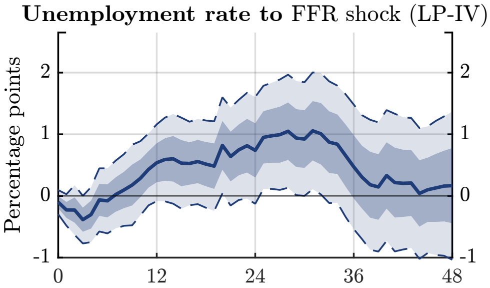
```
% LP-IV -- unemployment response to FFR shock (monthly)
LPopt_IV          = LPoption;
LPopt_IV.nsteps   = 49;          % 49-month horizon
LPopt_IV.longdiff = 1;           % LHS: urate_{t+h} - urate_{t-1}
LPopt_IV.impact   = 1;           % unit shock
LPopt_IV.pctg     = 95;
LPopt_IV.IV       = RRCGShock;
LPopt_IV.nlag_iv  = 6;           % lags of instrument
CTRL_IV = [urate infl ffr];
LP_IV   = LPmodel(urate, ffr, CTRL_IV, 6, 1, LPopt_IV);
IR_IV     = LP_IV.IR;
INF95_IV  = LP_IV.INF;
SUP95_IV  = LP_IV.SUP;
```

- `LPopt.IV` holds the **external instrument** (not the treatment); the endogenous treatment is the second argument to `LPmodel`

Results: Figures 5a and 6a

- LP-OLS: CPI falls by 2–4% cumulatively after a contractionary shock. LP-IV: unemployment rises, with the response building over 12–24 months





Appendix

References I

Antolín-Díaz, J. and J. F. Rubio-Ramírez (2018): "Narrative Sign Restrictions for SVARs," *American Economic Review*, 108, 2802–2829.

Blanchard, O. J. and D. Quah (1989): "The Dynamic Effects of Aggregate Demand and Supply Disturbances," *American Economic Review*, 79, 655–73.

Gertler, M. and P. Karadi (2015): "Monetary Policy Surprises, Credit Costs, and Economic Activity," *American Economic Journal: Macroeconomics*, 7, 44–76.

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