长安大学2021–2022学年第二学期
《计算方法》试题(A)卷参考答案

一、解：$\|A\|_\infty = 1.1, \|A\|_1 = 0.8, \|A\|_2 = 0.8279, \|A\|_F = 0.8426.$

二、解：Doolittle分解为

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}.$$

$$Ly = b \Rightarrow y = [5 \quad 3 \quad 6 \quad 4]^T; \quad Ux = y \Rightarrow x = [1 \quad 1 \quad 2 \quad 2]^T.$$

三、解：设$\phi_0(x) = \frac{1}{x}, \quad \phi_1(x) = x^2$，则正则方程组为

$$\begin{bmatrix} (\phi_0, \phi_0) & (\phi_0, \phi_1) \\ (\phi_1, \phi_0) & (\phi_1, \phi_1) \end{bmatrix} = \begin{bmatrix} (\phi_0, y) \\ (\phi_1, y) \end{bmatrix}$$

$$\text{即} \begin{bmatrix} 2.5 & 0 \\ 0 & 34 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 24.75 \\ 3.8 \end{bmatrix}$$

$$\text{解得：} y = \frac{9.9}{x} + 0.1118x^2.$$

四、证明：令$f(x) = x^k$，则基于节点$\{x_i\}_{i=0}^n$的n次拉格朗日插值多项式为$L_n(x) = \sum\limits_{i=0}^{n} f(x_i)l_i(x) = \sum\limits_{i=0}^{n} x_i^k l_i(x)$，当$k \le n$时，$R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}\omega_{n+1}(x) = 0$，所以$f(x) = L_n(x)$，结论成立.

五、解：(1)确定最大实根区间$[3,4]$; (2)建立迭代公式$x_{k+1} = \frac{1}{2}(\lg x_k + 7)$；(3)验证当$x \in [3,4]$时，$\phi(x) = \frac{1}{2}(\lg x + 7) \in [3,4]$; (4)当$x \in [3,4]$时有$|\phi'(x)| \le \phi'(3) \approx 0.07 < 1$,所以迭代法收敛；(5)取$x_0 = 4$，有$x_1 = 3.801030, x_2 = 3.789951, x_3 = 3.789317, x_4 = 3.789280$, x_4满足要求.

六、解：取$x_0 = -2, y_0 = -1, x_1 = 0, y_1 = 1, x_2 = 3, y_2 = -2, x_3 = 5, y_3 = 8$, 则Lagrange插值多项式为

$$L_3(x) = \sum_{i=0}^{3} y_i l_i(x)$$

$$l_0(x)=\frac{x(x-3)(x-5)}{-70},$$
$$l_1(x)=\frac{(x+2)(x-3)(x-5)}{30},$$
$$l_2(x)=\frac{x(x+2)(x-5)}{-30},$$
$$l_3(x)=\frac{x(x+2)(x-3)}{70},$$

差商表为

x_i	y_i	一阶差商	二阶差商	三阶差商
-2	-1			
0	1	1		
3	-2	-1	-2/5	
5	8	5	6/5	8/35

则Newton插值多项式为

$$N_3(x)=-1+(x+2)-\frac{2}{5}x(x+2)+\frac{8}{35}x(x+2)(x-3)$$
$$L_3(x)=N_3(x)=\frac{8}{35}x^3-\frac{22}{35}x^2-\frac{41}{35}x+1.$$

七、解：(1)

x	1	1.25	1.5	1.75	2
y	2.7183	3.4903	4.4817	5.7546	7.3891

(2) $I\approx 0.5/6*[f(1)+4f(5/4)+2f(3/2)+4f(7/4)+f(8)]=4.6708$

(3) $|R_S[f]|=\frac{b-a}{2880}h^4|f^{(4)}(\eta)|\le\frac{e^2}{2880n^4}\le 0.5*10^{-4}, n\ge 2.6764, \therefore n=3$，应至少取7个节点.

八、解：（1）雅克比迭代法迭代矩阵为 $B_J=\begin{bmatrix}0&-2&2\\-1&0&-1\\-2&-2&0\end{bmatrix}$,$|\lambda I-B_J|=\lambda^3, B_J$的特征值为$\lambda_{1,2,3}=0$，故 $\rho\left(B_J\right)=0<1$，雅克比法对任意初始向量都收敛.

(2) $x^{(1)}=[-3,1,1]^T$，$x^{(2)}=[-3,3,5]^T$，$x^{(1)}=[1,-1,1]^T$，$x^{(4)}=[1,-1,1]^T$.

(3) 高斯-赛德尔迭代法 $B_{G-S}=\begin{bmatrix}0&-2&2\\0&2&-3\\0&0&2\end{bmatrix}$，$|\lambda I-B_{G-S}|=\lambda(\lambda-2)^2$，$\lambda_1=0$，$\lambda_{2,3}=2$，$\rho\left(B_{G-S}\right)=2\ge 1$，高斯-赛德尔迭代法不是对任意初始向量均收敛.