APPLICATIONS OF MATLAB IN ENGINEERING

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Today:

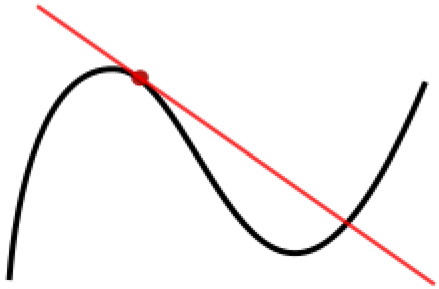
- Polynomial differentiation and integration
- Numerical differentiation and integration

Differentiation

- The derivative of a function $f(x)$ is written as

$$f'(x) \quad \text{or} \quad \frac{df(x)}{dx}$$

- The rate of the change in the function $f(x)$ with respect to x
- Geometrically, $f'(x_0)$ represents the coefficient of the line tangent to the curve in the point x_0



Polynomial Differentiation

- Polynomials are often used in numerical calculations
- For a polynomial

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

the derivative is

$$f'(x) = a_n n x^{n-1} + a_{n-1}(n-1) x^{n-2} + \cdots + a_1$$

- How do we calculate this using MATLAB?

Representing Polynomials in MATLAB

- Polynomials were represented as row vectors
- For example, consider the equation

$$f(x) = x^3 - 2x - 5$$

- To enter this polynomial into MATLAB, use

```
p = [1 0 -2 -5];
```

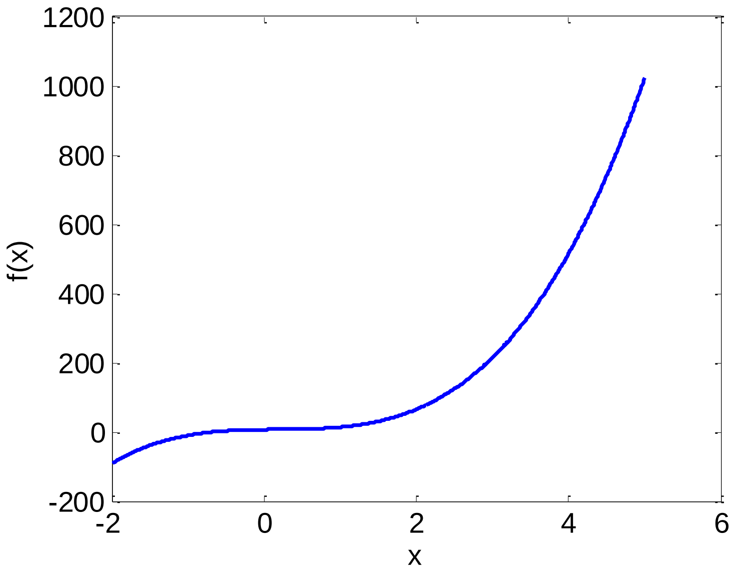
Values of Polynomials: `polyval()`

- Plot the polynomial

$$9x^3 - 5x^2 + 3x + 7$$

for $-2 \leq x \leq 5$

```
a = [9,-5,3,7];  x = -2:0.01:5;
f = polyval(a,x);
plot(x,f,'LineWidth', 2);
xlabel('x');  ylabel('f(x)');
set(gca, 'FontSize', 14)
```



Polynomial Differentiation: `polyder()`

- Given $f(x) = 5x^4 - 2x^2 + 1$
  1. What is the derivative of the function $f'(x)$?
  2. What is the derivative of the function value of $f'(7)$?

```
p=[5 0 -2 0 1];
polyder(p)
```

```
polyval(polyder(p),7)
```

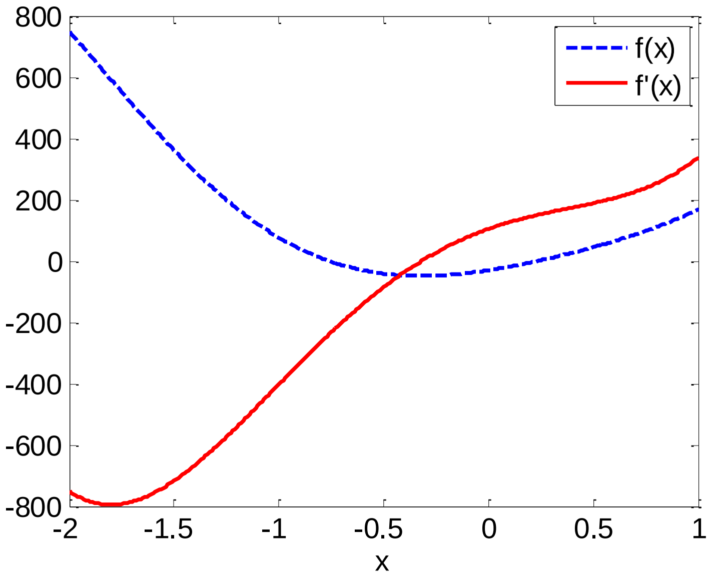
Exercise

- Plot the polynomial

$$f(x) = (5x^3 - 7x^2 + 5x + 10)(4x^2 + 12x - 3)$$

and its derivative for $-2 \leq x \leq 1$

- Hint: `conv()`



Polynomial Integration

- For a polynomial

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

the integration is

$$\int f(x) = \frac{1}{n+1} a_n x^{n+1} + \frac{1}{n} a_{n-1} x^n + \cdots + a_0 x + k$$

- How do we calculate this using MATLAB?

Polynomial Integration: `polyint()`

- Given $f(x) = 5x^4 - 2x^2 + 1$
  1. What is the integral of the function $\int f(x)$ with a constant of 3?
  2. What is the derivative of the function value of $\int f(7)$?

```
p=[5 0 -2 0 1];
polyint(p, 3)
```

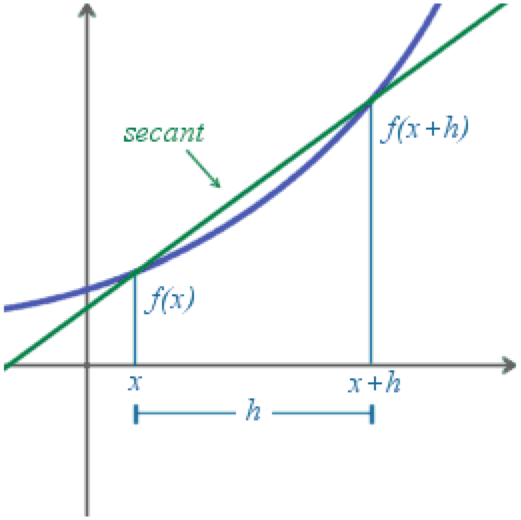
```
polyval(polyint(p, 3),7)
```

Numerical Differentiation

- The simplest method:
  finite difference approximation
- Calculating a secant line in the vicinity of x_0

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h},$$

where h represents a small change in x



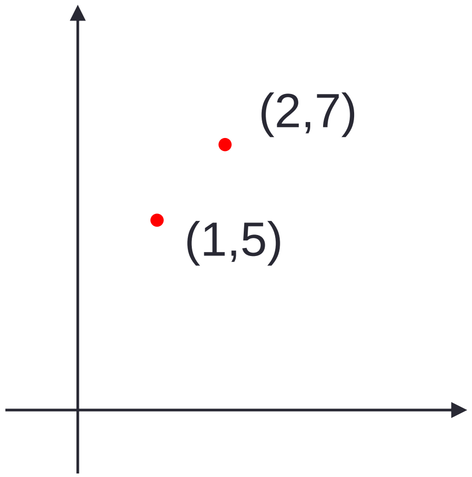
Differences: `diff()`

- `diff()` calculates the differences between adjacent elements of a vector

```
x = [1  2  5  2  1];
diff(x)
```

- Exercise: obtain the slope of a line between 2 points (1,5) and (2,7)

```
x = [1  2];  y = [5  7];
slope = diff(y)./diff(x)
```

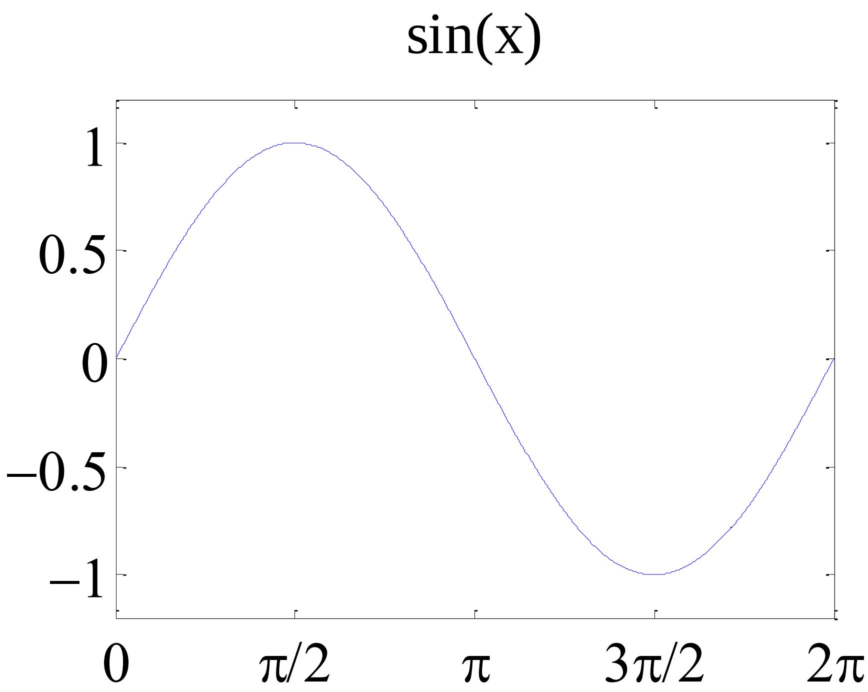


Numerical Differentiation Using `diff()`

- Given $f(x) = \sin(x)$, find $f'(x_0)$ at $x_0 = \pi/2$ using $h = 0.1$

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

```
x0 = pi/2;   h = 0.1;
x = [x0 x0+h];
y = [sin(x0) sin(x0+h)];
m = diff(y)./diff(x)
```



- How does h affect accuracy?

Exercise

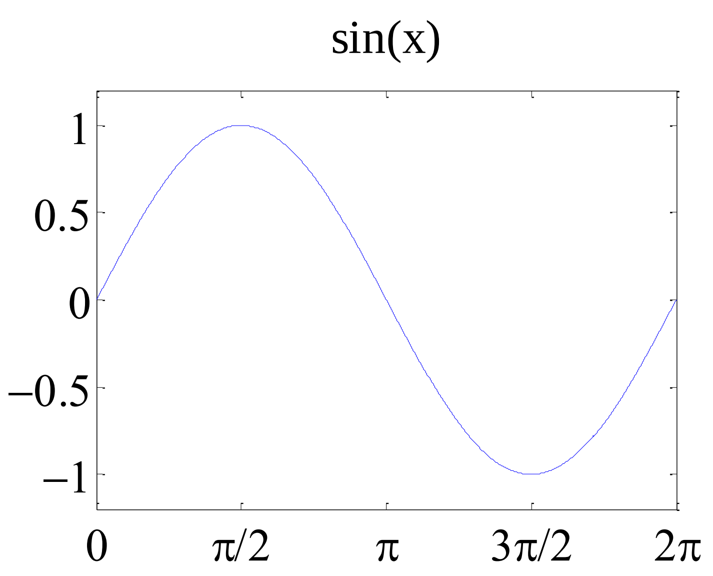
- Given $f(x) = \sin(x)$, write a script to find the error of $f'(x_0)$ at $x_0 = \pi/2$ using various h

h	Error of $f'(x_0)$
0.1	
0.01	
0.001	
0.0001	
0.00001	
0.000001	
0.0000001	

How to Find the f' over An Interval $[0, 2\pi]$?

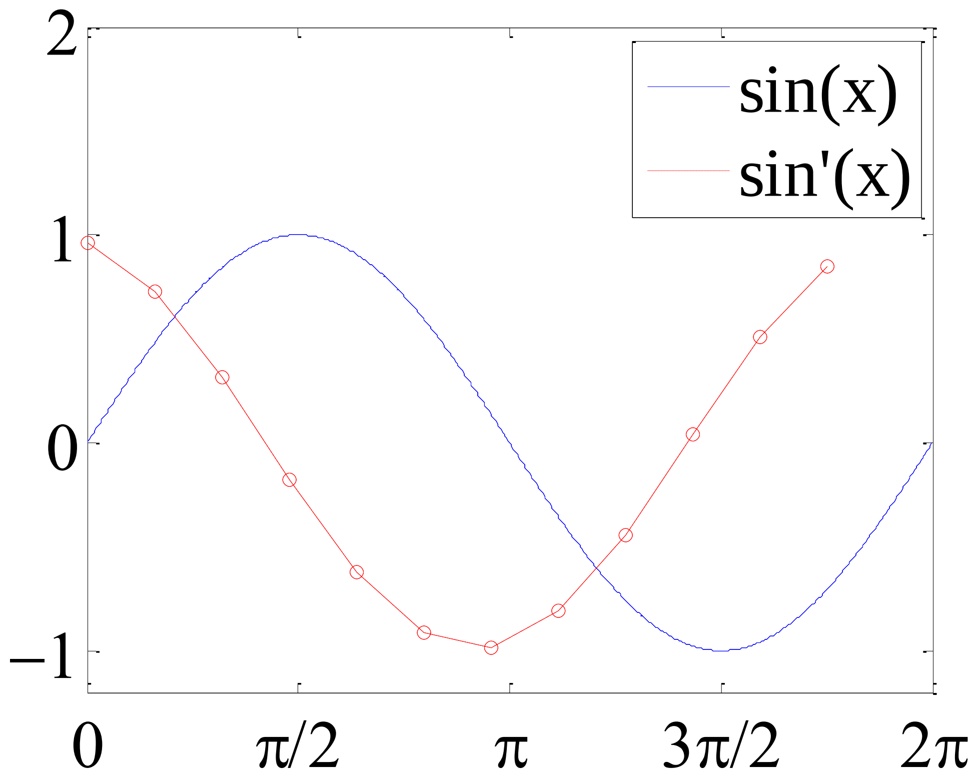
- In the previous example, $x_0 = \pi/2$
- Strategy:
  1. Create an array in the interval $[0, 2\pi]$
  2. The step is the h
  3. Calculate the f' at these points
- For example,

```
h = 0.5;  x = 0:h:2*pi;
```



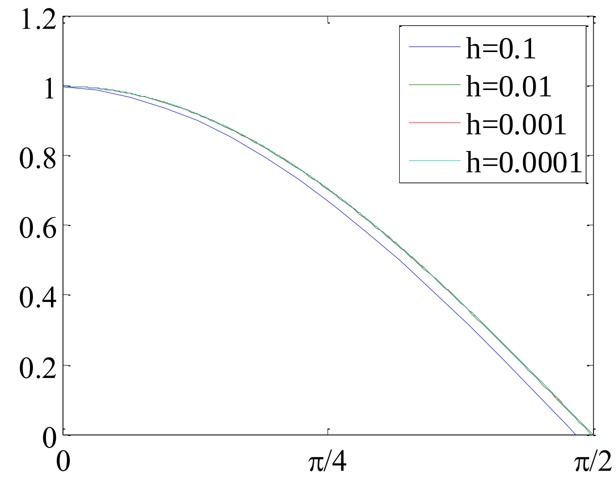
Find $\sin'(x)$ over $x = [0, 2\pi]$

```
h = 0.5;   x = 0:h:2*pi;
y = sin(x);   m = diff(y)./diff(x);
```



Various Step Size

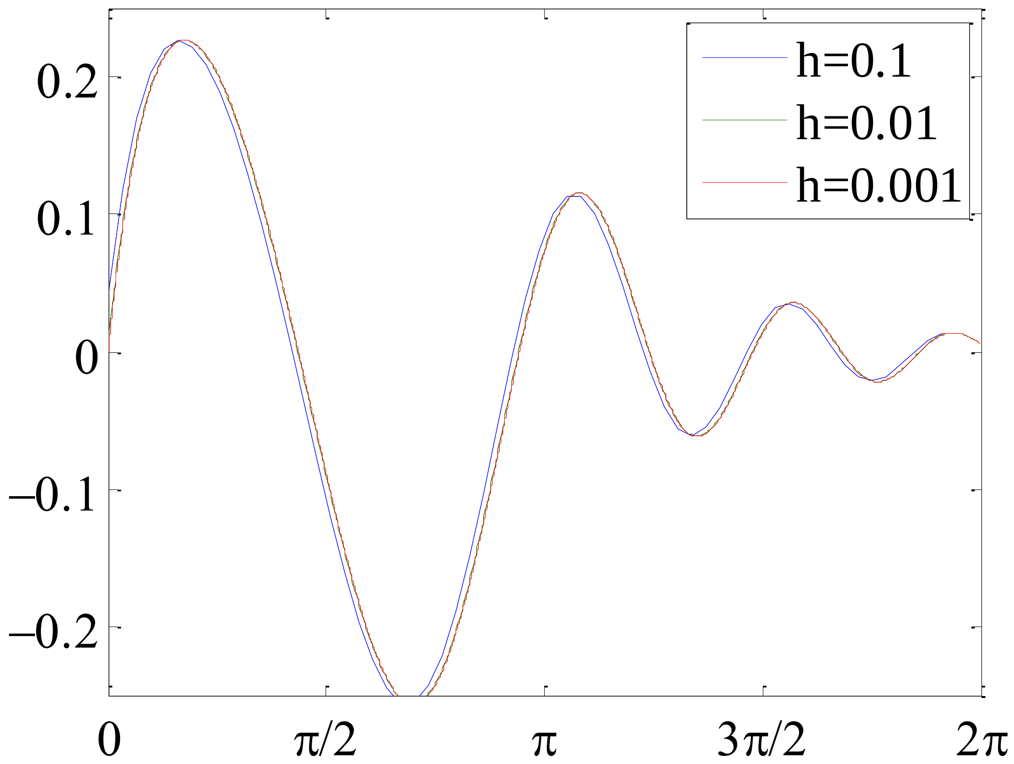
- The derivatives of $f(x) = \sin(x)$ calculated using various h values




```
g = colormap(lines);  hold on;
for i=1:4
    x = 0:power(10, -i):pi;
    y = sin(x);  m = diff(y)./diff(x);
    plot(x(1:end-1), m, 'Color', g(i,:));
end
hold off;
set(gca, 'XLim', [0, pi/2]);  set(gca, 'YLim', [0, 1.2]);
set(gca, 'FontSize', 18);  set(gca, 'FontName', 'symbol');
set(gca, 'XTick', 0:pi/4:pi/2);
set(gca, 'XTickLabel', {'0', 'p/4', 'p/2'});
h = legend('h=0.1','h=0.01','h=0.001','h=0.0001');
set(h,'FontName', 'Times New Roman');  box on;
```

Exercise

- Given $f(x) = e^{-x}\sin(x^2/2)$, plot the approximate derivatives f' of $h = 0.1$, 0.01, and 0.001

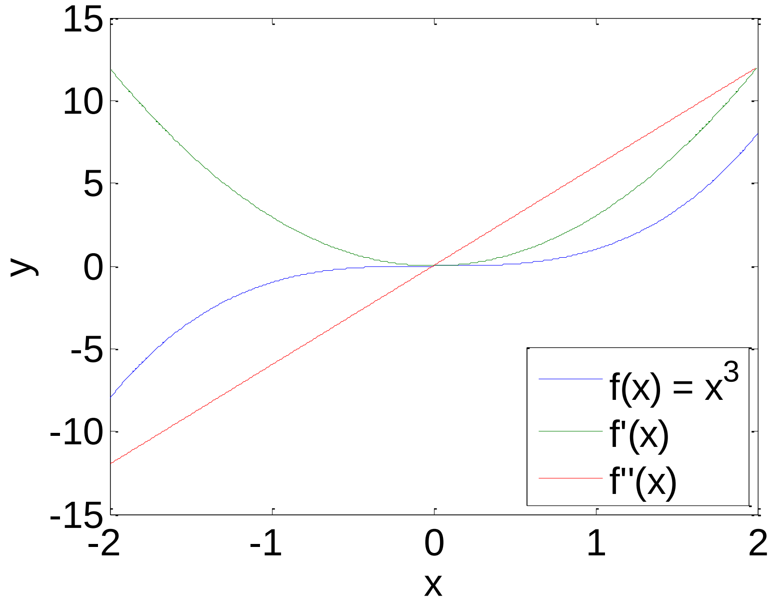


Second and Third Derivatives

- The second derivative f'' and third derivative f''' can be obtained using similar approaches
- Given $f(x) = x^3$, plot f' and f' for $-2 \leq x \leq 2$

```
x = -2:0.005:2;  y = x.^3;
m = diff(y)./diff(x);
m2 = diff(m)./diff(x(1:end-1));

plot(x,y,x(1:end-1),m,x(1:end-2),m2);
xlabel('x', 'FontSize', 18);
ylabel('y', 'FontSize', 18);
legend('f(x) =
x^3','f''(x)','f''''(x)', 4);
set(gca, 'FontSize', 18);
```

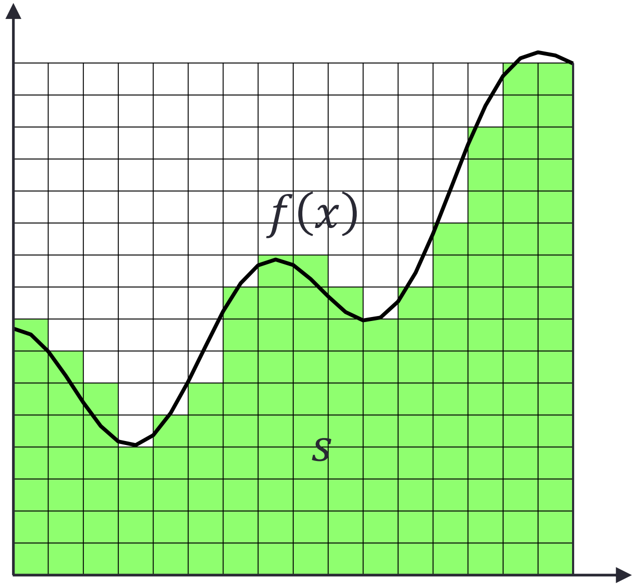


Numerical Integration

- Calculating the numerical value of a definite integral

$$s=\int_a^b f(x)d(x)\approx\sum_{i=0}^{n} f(x_i)\int_a^b L_i(x)dx$$

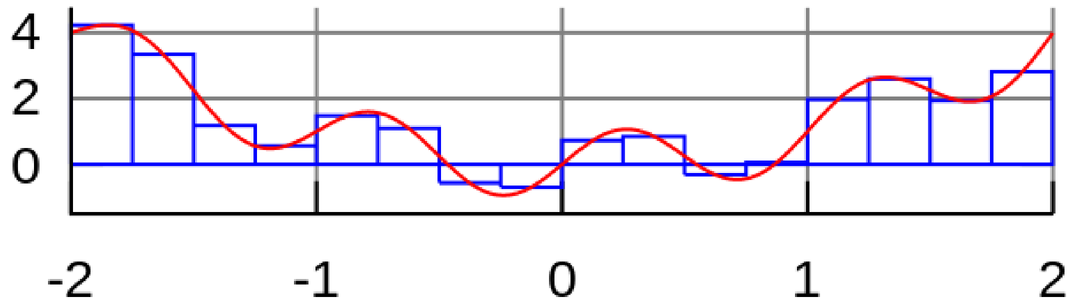
- Quadrature method – approximating the integral by using a finite set of points



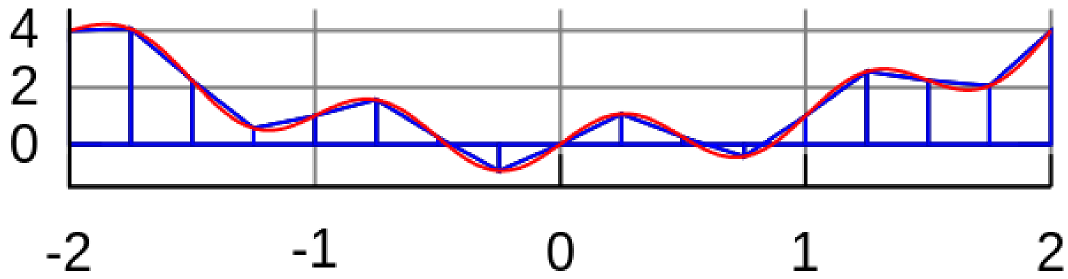
Numerical Quadrature Rules

- Basic quadrature rules:

1. Midpoint rule (zeroth-order approximation)

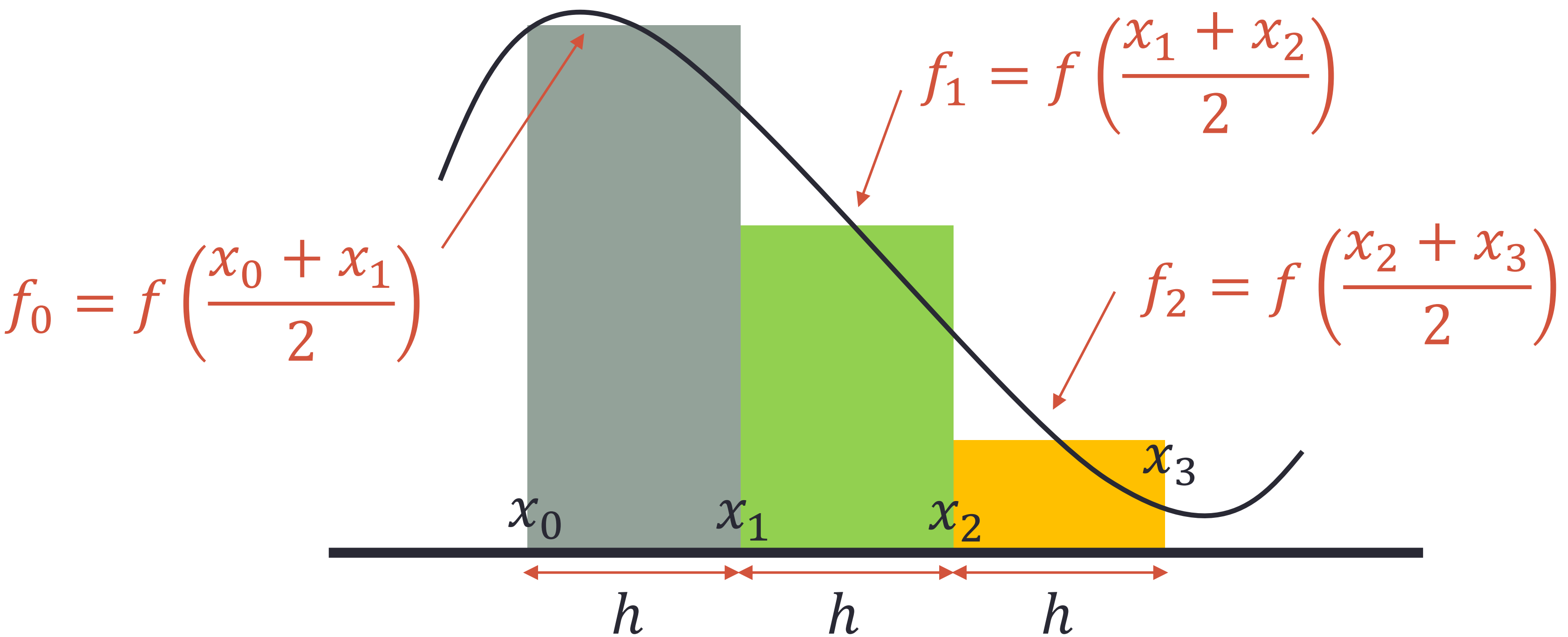


2. Trapezoid rule (first-order approximation)



Midpoint Rule

$$\int_{x_0}^{x_3} f(x)dx \approx hf_0 + hf_1 + hf_2 = h\sum_{i=0}^{n-1} f_i$$



Midpoint Rule Using `sum()`

- Example:

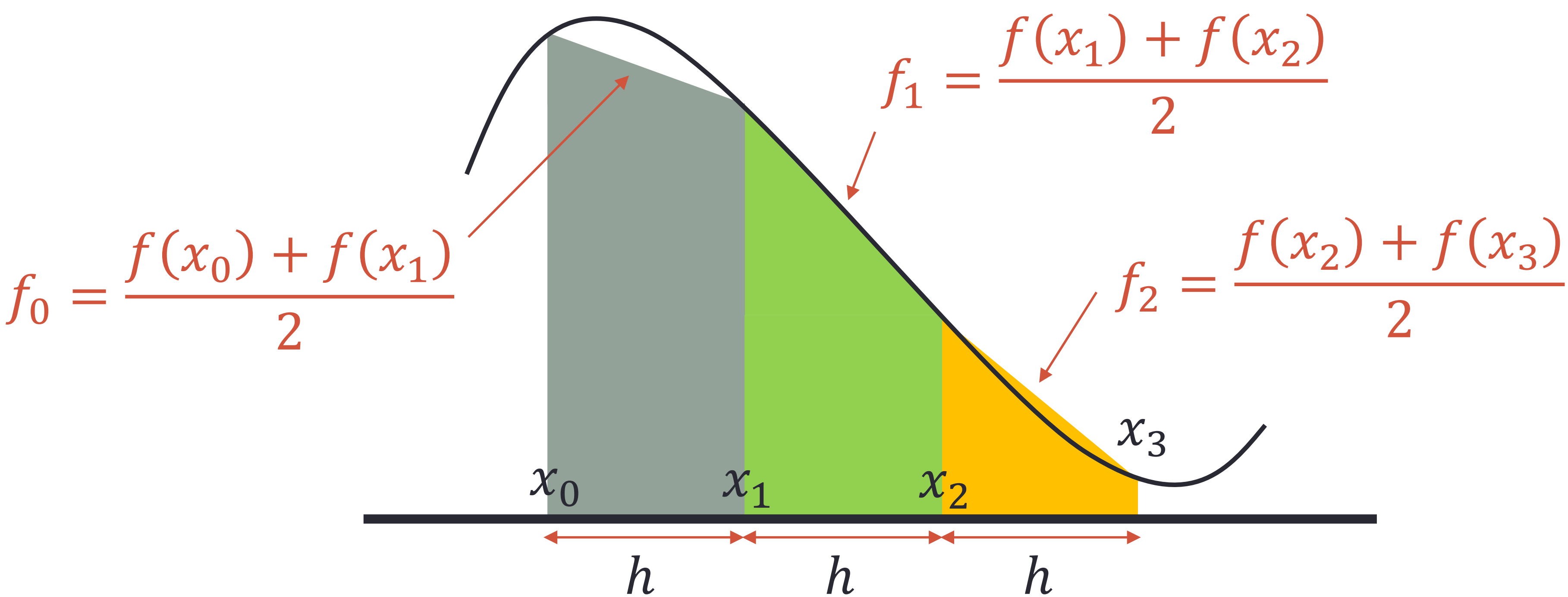
$$A = \int_0^2 4x^3\,dx = x^4|_0^2 = (2)^4 - (0)^4 = 16$$

```
h = 0.05;  x = 0:h:2;
midpoint = (x(1:end-1)+x(2:end))./2;
y = 4*midpoint.^3;
s = sum(h*y)
```

- How accurate is it?
- How to improve the accuracy?

Trapezoid Rule

$$\int_{x_0}^{x_3} f(x)dx \approx h\frac{f_0 + f_1}{2} + h\frac{f_1 + f_2}{2} + h\frac{f_2 + f_3}{2}$$



Trapezoid Rule Using `trapz()`

- Example:

$$A = \int_0^2 4x^3\,dx = x^4|_0^2 = (2)^4 - (0)^4 = 16$$

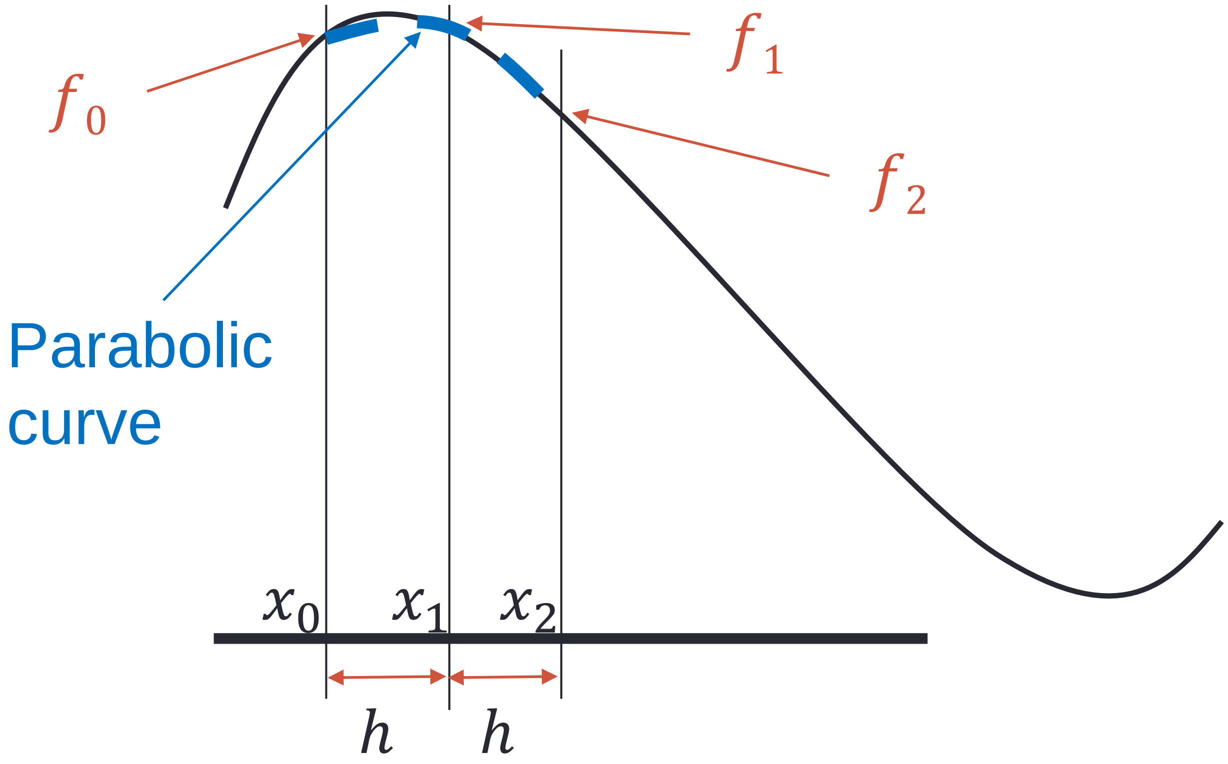
```
h = 0.05; x = 0:h:2; y = 4*x.^3;
s = h*trapz(y)
```

- How accurate is it?
- Alternative:

```
h = 0.05;  x = 0:h:2;  y = 4*x.^3;
trapezoid = (y(1:end-1)+y(2:end))/2;
s = h*sum(trapezoid)
```

Second-order Rule: $\frac{1}{3}$ Simpson's

$$\int_{x_0}^{x_2} f(x)dx \approx \frac{h}{3}(f_0 + 4f_1 + f_2)$$



Simpson's Rule

- Example:

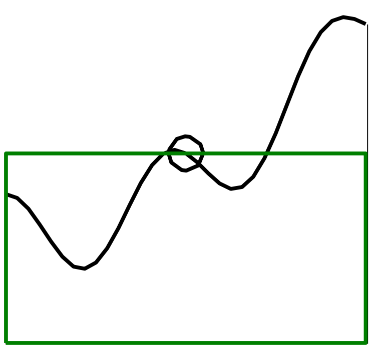
$$A = \int_0^2 4x^3\,dx = x^4|_0^2 = (2)^4 - (0)^4 = 16$$

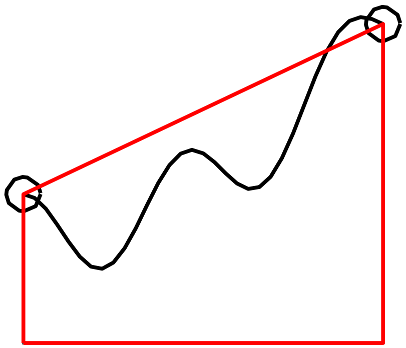
```
h = 0.05;  x = 0:h:2;  y = 4*x.^3;
s = h/3*(y(1)+2*sum(y(3:2:end-2))+...
    4*sum(y(2:2:end))+y(end))
```

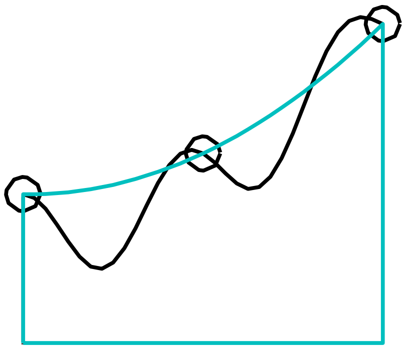
- How accurate is it?

Comparison

Name	Degree	Formula
Midpoint Rule	0	$\int_a^b f(x)dx = 2h \sum_{i=0}^{(n/2)-1} f(x_{2i+1})$
Trapezoid Rule	1	$\int_a^b f(x)dx = \frac{h}{2}\left[f(a) + 2\sum_{i=1}^{n-1} f(x_i) + f(b)\right]$
Simpson's Rule	2	$\int_a^b f(x)dx = \frac{h}{3}\left[f(a) + 2\sum_{i=1}^{(n/2)-1} f(x_{2i}) + 4\sum_{i=1}^{n/2} f(x_{2i-1}) + f(b)\right]$


Midpoint rule


Trapezoid rule


Simpson's rule

Review of Function Handles (`@`)

- A handle is "a pointer to a function"
- Can be used to pass functions to other functions
- Example:

Pass a function `f(x)=sin(x)`
to a user-defined function: `g(f,…)`

```
f=sin(x)

g(@f,…)
```

Function Handles (@) Example

- The `input` of the following function `xy_plot` is a math function:

```
function [y] = xy_plot(input,x)
% xy_plot receives the handle of a function
% and plots that function of x
y = input(x);  plot(x,y,'r--');
xlabel('x');  ylabel('function(x)');
end
```

- Try:

```
xy_plot(@sin,0:0.01:2*pi);
xy_plot(@cos,0:0.01:2*pi);
xy_plot(@exp,0:0.01:2*pi);
```

Numerical Integration: `integral()`

- Numerical integration on a function from using global adaptive quadrature and default error tolerances

- Example: $\int_{0}^{2} \frac{1}{x^3-2x-5}\,dx$

```
y = @(x) 1./(x.^3-2*x-5);
integral(y,0,2)
```

Double and Triple Integrals

- Example $f(x,y) = \int_0^{\pi} \int_{\pi}^{2\pi} (y \cdot \sin(x) + x \cdot \cos(y))dx\,dy$

```
f = @(x,y) y.*sin(x)+x.*cos(y);
integral2(f,pi,2*pi,0,pi)
```

- Example: $f(x,y) = \int_{-1}^{1} \int_0^{1} \int_0^{\pi} (y \cdot \sin(x) + z \cdot \cos(y))dx\,dy\,dz$

```
f = @(x,y,z) y.*sin(x)+z.*cos(y);
integral3(f,0,pi,0,1,-1,1)
```

End of Class